

# Analysis of a Modified OFDM System Minimizing Frequency Offset by Dividing Subchannels

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**Abstract**—This paper reviews and introduces a modified CPD-OFDM (Orthogonal Frequency Division Multiplexing with cross polarization diversity structure) system improving the system performance degradation due to the frequency offset. In the conventional OFDM system, the frequency offset causes the ICI (inter-channel interference) and degrades signal-to-noise ratio in the receiving end. The cross polarization diversity structure composed with 2-pairs cross polarized circular antenna in each transceiver has the characteristic that it can remarkably remove the odd time reflected waves in each receiving end. Because of this receiving effect for removing the delayed multipath waves, the cross circular polarization diversity structure can reduce the time delay spread and ICI. Therefore, the modified CPD-OFDM system can improve the system performance as well as spectrum efficiency. In order to investigate the performance degradation of CPD-OFDM system due to the frequency offset, computer simulation and theoretical analysis were conducted. From the simulation results, it is clearly seen that the CPD-OFDM system performance has shown 1~3 [dB] improvement compared to that of the conventional OFDM system.

**Keywords**—Modified CPD-OFDM; diversity structure; divides sub-channel; minimizing frequency offset; improving performance

## I. INTRODUCTION

Many researches on high quality digital transmission system is still eagerly conducted to realize the broadband multimedia communication system with more high speed. Usually, OFDM system generates many subcarriers in each parallel sub-channels. Because of this parallel transmission structure of the long interval OFDM symbols, inter symbol interference between each adjacent channels can be easily diminished [1]. And also, because of orthogonal signal configuration in OFDM system, system bandwidth has been a decrease [2]. For the aim of orthogonality between the subcarriers, the OFDM adopts cyclic prefix in guard channel data frame. In general, in order to sustaining the orthogonality between subcarrier signals, guard channel data frame should be chosen more larger than the time delay spread of radio channel [3]. In case of the cyclic prefix is set larger than the time delay of radio channel, it keeps the orthogonality between sub-channels [4]. However, in case of the delay time of multipath rays becomes more longer than the time of guard channel frames, it makes channel interference and then causes system performance degradation [5].

We reviews and introduces a modified OFDM system compose of cross polarization diversity structure (CPD-OFDM) for reducing the frequency offset occurred in conventional OFDM system. By adopting the polarization diversity structure with 2-pairs circular polarized antenna in conventional OFDM systems, both the time delay spread and ICI can be minimized. From the analysis results of system performance, it was shown that the modified CPD-OFDM system has been a better performance compared to the OFDM system without polarization diversity structures.

## II. THE CONVENTIONAL OFDM SYSTEM

In OFDM system, the  $k^{\text{th}}$  subcarrier  $f_k$  is defined by (1):

$$f_k = \frac{k}{NT_{ds}} = \frac{k}{T_s} \quad (1)$$

Where,  $N$  is the integer number of subcarriers,  $T_{ds}$  is symbol data duration and  $T_s$  is OFDM symbol duration. Therefore, the baseband signal of the OFDM  $s(t)$  is expressed as follows [6]:

$$s(t) = \sum_{n=-\infty}^{\infty} \sum_{k=0}^{N-1} \frac{C}{\sqrt{T_s}} d_{n,k} e^{j2\pi f_k t} p(t - nT_s) \quad (2)$$

In (3),  $C$  is the constant related to the power of the signal, and  $d_{n,k}$  is symbol data allocated to  $k^{\text{th}}$  subcarrier between the  $n^{\text{th}}$  symbol duration  $[nT_s, (n+1)T_s]$ . The  $p(t)$  represents pulse shaping function. An OFDM frame signal  $s_i(t)$  in the period of  $[nT_s, (n+1)T_s]$  is given by

$$s_i(t) = \sum_{k=0}^{N-1} \frac{C}{\sqrt{T_s}} d_{n,k} e^{j2\pi f_k t} \quad (3)$$

where,  $nT_s \leq t < (n+1)T_s$ . Therefore, the sampled signal of the OFDM frame signal  $s(m)$  is expressed as follows:

$$s(m) = \sum_{k=0}^{N-1} \frac{C}{\sqrt{T_s}} d_{n,k} e^{j2\pi km/N} \quad (4)$$

$$\text{where, } f_k = \frac{k}{NT_{ds}}, \quad m = 0, 1, \dots, N-1.$$

This (4) has the same results that can be obtained from inverse discrete Fourier transform (IDFT) for the symbol data  $d_{n,k}$ . It means that the modulation of the subcarrier can be achieved by the IDFT for the data symbol. All data symbols in the subcarriers can be transmitted simultaneously using the orthogonality between subcarriers if there is no distortion in the channel. In order to eliminate ICI, the guard interval should be inserted in each OFDM frame. This guard interval should be kept longer than the time delay spread of radio channel [5]. Usually, in the case of the time delay spread is longer than the guard channel data interval, the OFDM system performance cannot be improved even though the SNR (signal to noise ratio) has been increased. From this reason, the orthogonality between the subcarriers cannot be sustained and then the ICI can be easily induced.

### III. CROSS POLARIZATION DIVERSITY

#### A. Basic Concept

The cross polarization diversity structure is composed of other phase difference using vertical polarization and horizon polarization. It also divides the sub-channels. The sub-channels is separated from 2-channels of RHCP (right-handed circular polarization) and LHCP (left-handed circular polarization), which has opposite polarized wave direction in each channel. The phase of vertical polarization wave is not change when it is reflected from the range off Brewster angle, but horizon polarization makes always occurrence of 180 degree phase inversion. Because of the circular polarization antenna is composed with 90 degree transition for the vertical and horizontal antenna configuration, RHCP change to LHCP when it is odd-time reflection, vice versa [7], [8]. It is also well known that the polarization diversity can reduce the interference of reflected waves and minimizes the multipath fading in indoor/outdoor wireless channel [9].

#### B. CPD-OFDM System [10]

In general, in OFDM system without the polarization diversity structure, because of sub-channel overlapping which do not keep the orthogonality between sub-carriers the frequency offset is easily occurred. This frequency offset effects can induces the system performance degradation. For the aim of minimizing frequency offset caused from the subcarriers overlapping, we adopt the above mentioned cross polarization diversity structure. The system configuration of CPD-OFDM is shown in Fig. 1. Fig. 2 shows a sub-carrier spectrum of CPD-OFDM system.

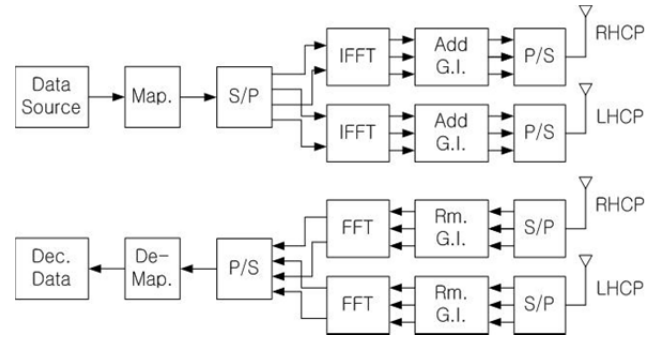


Fig. 1. Configuration of CPD-OFDM system.

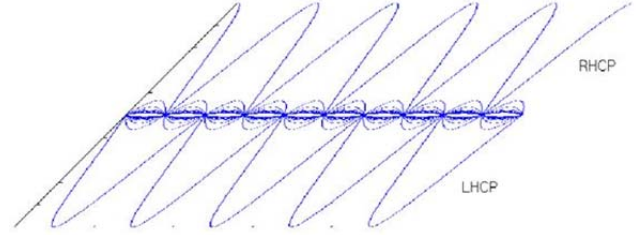


Fig. 2. Subcarrier spectrum of CPD-OFDM system.

The CPD-OFDM system separates the sub-channels into a RHCP channel and a LHCP channel, where they have opposite circularly polarized waves individually. The signals of RHCP channel  $s_R(t)$  and the signal of the LHCP channel  $s_L(t)$  can be achieved from (2) as follows:

$$s_R(t) = \sum_{n=-\infty}^{\infty} \sum_{k=0}^{(N-1)/2} \frac{C}{\sqrt{T_s}} d_{n,(2k)} e^{j2\pi f_{(2k)} t} p(t-nT_s) \quad (5)$$

$$s_L(t) = \sum_{n=-\infty}^{\infty} \sum_{k=0}^{(N-1)/2} \frac{C}{\sqrt{T_s}} d_{n,(2k+1)} e^{j2\pi f_{(2k+1)} t} p(t-nT_s) \quad (6)$$

Fig. 3 shows the channel separation of CPD-OFDM system. In this figure,  $h_{ij}$  is the transfer function that has the characteristics of transmission in the  $i^{\text{th}}$  channel and reception by the  $j^{\text{th}}$  channel.

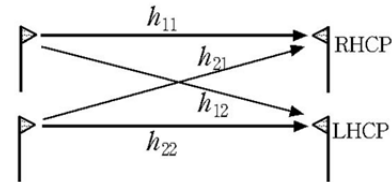


Fig. 3. Channel separation of CPD-OFDM system.

Therefore, the received signal in each channel of CPD-OFDM system is given by

$$r_R(t) = h_{11}s_R(t) + \Gamma_c h_{21}s_L(t) + N_R(t) \quad (7)$$

$$r_L(t) = h_{22}s_L(t) + \Gamma_c h_{12}s_R(t) + N_L(t) \quad (8)$$

Where,  $N(t)$  is the AWGN and  $\Gamma_c$  is polarization discrimination ratio to the ordinary transmitted circularly polarized wave and it can be written as follow:

$$\Gamma_c = 20 \log_{10} \left| \frac{E_{RL}}{E_{RR}} \right| = 20 \log_{10} \left| \frac{E_{LR}}{E_{LL}} \right| \quad (9)$$

In (9),  $E_{RR}$  means the strength of the electric fields which is transmitted in the RHCP channel and received by the RHCP channel and so on.

#### IV. CPD-OFDM SYSTEM ANALYSIS

##### A. Time Delay Effect in Conventional OFDM System

The time delay of channel occurs interference and loss of orthogonality, and then performance degradation. Fig. 4 shows BER performance of OFDM system according to the time delay increase in each case of SNR value [11]. It shows that BER performance more degrades when the delay time of reflection wave becomes more longer. Therefore, guard interval is extended to reduce a bad influence induced by the time delay of the reflection waves. However, guard interval extension caused the degradation of transmission efficiency because of undesirable data stream interval. For this reason, it can be convinced that the CPD-OFDM using cross circular polarization wave improved bandwidth efficiency and transmission efficiency.

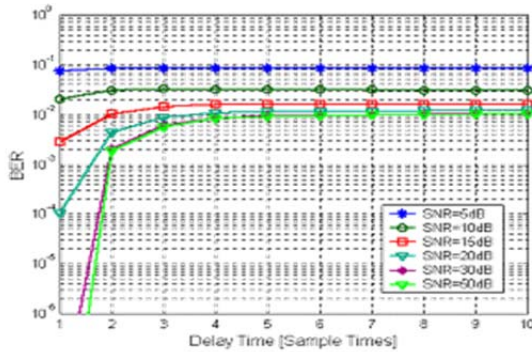


Fig. 4. Simulation results of OFDM related to the time delay spread scale [11].

##### B. Time Delay Spread

If impulse  $\delta(t)$  is transmitted at time  $t = 0$ , received signal  $r(t)$  is defined by

$$r(t) = \sum_{i=1}^n A_i \delta(t - T_i) \quad (10)$$

$n$  is the integer number of signal data streams,  $A_i$  is the magnitude of the received impulse at the  $i^{\text{th}}$  stream and  $T_i$  is

the time delay of the  $i^{\text{th}}$  stream. In this case, the impulse response of delay time  $T$  shows its characteristic by probability density function  $p(T)$ , and delay variance  $\sigma_T$ .

$$p(T) = \frac{1}{T} \exp(-T/\bar{T}) \quad (11)$$

$$\sigma_T = \sqrt{E[T^2] - E^2[T]} \quad (12)$$

In (11),  $\bar{T} = E[T] = \int_0^{\infty} T p(T) dT$ . These (11) and (12)

show that time delay spread  $\sigma_T$  which is reduced by receiving impulse with delay time  $T$ , and shows that probability density function is decreased by  $p(T)$ . Therefore, the time delay spread  $\sigma_T$  for the channel is reduced according to time delay  $T$  and probability density function  $p(T)$  which can be also reduced by the removing effect of odd-time reflected waves in cross polarization diversity reception. This is because the time delay spread can be minimized by removing the odd-time reflection waves in circularly polarized wave.

Time delay spread is about 20 ns on 4~6 GHz in the indoor channel. But, in circularly polarized wave, time delay spread can be much less than that of vertical or horizon polarization by 4.5ns. This kind of delay time is much shorter than sampling period of OFDM. This means that the time interval of guard channel data frame has been a decrease by adopting circularly polarized wave.

##### C. System Bandwidth Efficiency

Transmission rate  $R$  and bandwidth  $W$  of OFDM system with M-PSK (Multi-Phase Shift Keying) modulation scheme is given by

$$R = \log_2 M \times 1 / NT_s \times N \quad (13)$$

$$W = f_{N-1} - f_0 + 2\sigma = (N - 1) / NT_s + 2\delta \quad (14)$$

Therefore, the bandwidth efficiency  $\eta$  is as follow [6]:

$$\eta = \frac{R}{W} = \frac{\log_2 M}{(1 - 1/N)T_s + 2\delta T_s + GI} \quad (15)$$

In (14),  $\delta$  represents single Bandwidth of subcarrier, it is  $\delta = (1 + \alpha) / 2NT_s$ . The  $\alpha$  represents roll-off rate and GI is the bandwidth of guard interval. It can be seen that substantial efficient bandwidth is increased by reducing length of the guard interval from the above equation. Usually, the length of guard interval can be up to about 25% in whole frame. It will be removed after the frame synchronization. Thus the bandwidth efficiency can be increased if the length of guard interval is reduced by cross polarization diversity.

## V. SIMULATION RESULTS

The computer simulation was conducted with the parameters of the IEEE 802.11a [12] by Monte Carlo simulation. The size of the FFT and IFFT are 64 points and sampling rates are 50ns. The total bandwidth is 20MHz and the mapping method is QPSK. The convolutional code scheme is adopted. The constraint length was 7 and code rate was 1/2. Thus, the data rate is 12Mbps. Multipath channel with 6-rays including direct wave was considered. Each reflected waves has the time interval of one sampling period, and the attenuation loss to the each reflected action were set in 3 [dB].

When the frequency offset is presented, both the system performance OFDM and CPD-OFDM with cross polarization diversity structure are estimated. Fig. 5 shows the SER (Symbol Error Rate) performance results of the OFDM which has frequency offsets. From Fig. 5, it is shown that the SER performance is remarkably degraded according to increasing the frequency offset. In the case of exceeding the 10 percent frequency offset it shows error floor. Fig. 6 shows comparisons of SER performance between the CPD-OFDM and the OFDM without polarization diversity scheme. From Fig. 6, the system performance of CPD-OFDM has been improved by 1~3 [dB] compared to that of the OFDM system without polarization diversity scheme.

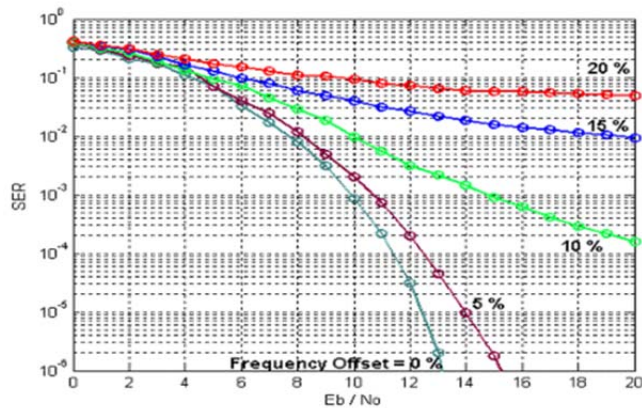


Fig. 5. Simulation results of SER.

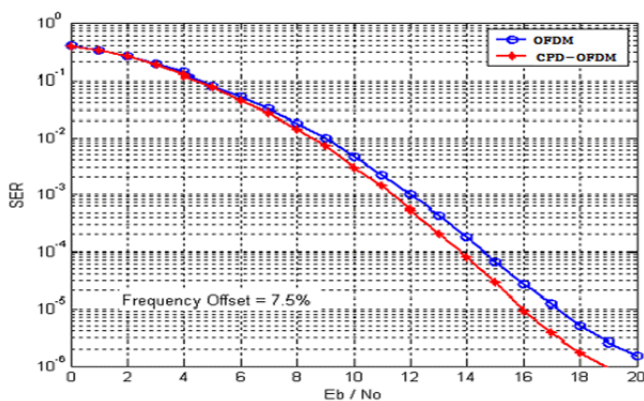


Fig. 6. Comparisons of SER performance (frequency offset is 7.5%).

## VI. CONCLUSIONS

We reviewed and introduced a modified CPD-OFDM system improving system performance degradation occurred from frequency offset. The CPD-OFDM system has the structure of dividing sub-channels into a RHCP and a LHCP channel. Therefore the orthogonality between sub-channels can be improved by eliminating the overlap of subcarriers using by cross polarization diversity structure. Because of the time delay spread can be minimized and the bandwidth efficiency can be also improved by this cross polarization diversity structure, the CPD-OFDM system shows that the performance degradation occurred from ICI inducing by frequency offset has been a remarkable decrease. It was also found that error floor occurs when frequency offset occur over 10% in OFDM system. However, the CPD-OFDM system minimizes time delay spread, and then the SER performance is improved by about 1~3[dB].

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