# Sea Ice Concentration Estimation Method with Satellite Based Visible to Near Infrared Radiometer Data Based on Category Decomposition

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Abstract—Unmixing method for estimation of mixing ratio of the components of which the pixel in concern consists based on inversion theory is proposed together with its application to sea ice estimation method with satellite based visible to near infrared radiometer data. Through comparative study on the different unmixing methods with remote sensing satellite imagery data, it is found that the proposed inversion theory based unmixing method is superior to the other methods. Also it is found that the proposed unmixing method is applicable to sea ice concentration estimations.

Keywords—Unmixing; Inversion theory; Category decomposition; Remote Sensing

#### I. INTRODUCTION

Estimation of mixing ratio of the components of which the pixel in concern consists is called as "Unmixing". Inversion theory based method is proposed together with its application to sea ice estimation method with satellite based visible to near infrared radiometer data. There are previously proposed methods such application oriented unmixing methods for hyperspectral instrument data, inversion theory based methods, etc.

On the other hands, there are many ice concentration estimation methods. Most of the methods utilize relations between brightness temperature and ice concentration. Estimation accuracy of the methods, however, is not good enough. In order to improve ice concentration estimation accuracy, inversion theory based unmixing method is introduced.

Through comparative study on the different unmixing methods with remote sensing satellite imagery data, it is found that the proposed inversion theory based unmixing method is superior to the other methods. Also it is found that the proposed unmixing method is applicable to sea ice concentration estimations.

The following section describes the proposed ice concentration estimation method followed by some experiments. Then conclusion is described with some discussions.

## II. PROPOSED SEA ICE CONCENTRATION ESTIMATION METHOD

### A. Conventional Method

The conventional method for sea ice concentration estimation method uses relation between brightness temperature which is acquired with microwave scanning radiometer onboard remote sensing satellite,  $T_B$  and sea ice concentration, *C*. Figure 1 shows an example of the relation. US Navy proposed this relation for sea ice concentration estimation. It is assumed empirically that brightness temperature of sea water is around 135K while that of sea ice is around 240K which is derived from NIMBUS-5/ESMR.



BRIGHTNESS TEMPERATURE

Fig. 1. Relation between brightness temperature and sea ice concentration

Glosen et al., proposed the equation (1) for sea ice concentration.

$$C = \frac{T_b - 135}{\varepsilon T_s - 135} \tag{1}$$

Where *Tb* and *Ts* denotes brightness temperature of sea ice and the ocean derived from NIMUS-7/ESMR and  $\varepsilon$  denotes emissivity of sea ice (around 0.92). NASDA (Current JAXA) proposed the following equation for sea ice concentration estimation,

$$C = 4.17D - 220.83 \tag{2}$$

Where *D* denotes digital number which corresponds to the brightness temperature derived from vertical polarization of 37 GHz of frequency channel of MOS-1/MSR (Marine Observation Satellite-1/ Microwave Scanning Radiometer).

Meanwhile, NASA Team (SSIAWT) proposed the following method with horizontal and vertical polarization of 19 GHz channels and vertical polarization of 37GHz channel of digital count of data.

$$CT = CF + CM$$

$$CF = \frac{(C_1 + C_2PR + C_3GR + C_4PR * GR)}{D}$$

$$CM = \frac{(C_9 + C_{10}PR + C_{11}GR + C_{12}PR * GR)}{D}$$

$$D = C_5 + C_6PR + C_7GR + C_8PR * GR$$

$$PR = \frac{TB(19, V) - TB(19, H)}{TB(19, V) + TB(19, H)}$$

$$GR = \frac{TB(37, V) - TB(19, V)}{TB(37, V) + TB(19, V)}$$
(3)

Where CT, CF, CM denotes total sea ice concentration, first year ice concentration, multiyear sea ice concentration. PR, GR denotes Polarization Ratio and Gradient Ratio, and D denotes digital number. Water vapor content in the atmosphere may affect to sea ice concentration estimations. In order to remove the influence due to water vapor, the following equation is utilizing,

$$GR > 0.05$$
 (4)

Then the following Comisso Bootstrap algorithm is widely used method for sea ice concentration estimation.

$$IC = \frac{\sqrt{(T_{B1} - T_{B1}^W)^2 + (T_{B2} - T_{B2}^W)^2}}{\sqrt{(T_{B1}^I - T_{B1}^W)^2 + (T_{B2}^I - T_{B2}^W)^2}}$$
(5)

The Bootstrap algorithm takes into account ocean area dependency (Antarctic and Arctic ocean areas are different treatments). Also open water is detected with the following equation,

$$WSLOPE * T_{B19V} + WINTRC > T_{B37V} \tag{6}$$

Seasonal changes and ocean area dependency are taken into account.

### B. Inversion Theory Based Unmixing Method

Mixing ratio of component of which the observed pixel consists of the components,  $a_i$  can be estimated as follows,

$$P = \sum_{j=1}^{k} a_j M_j$$
$$\sum_{j=1}^{k} a_j = 1$$
$$a_j \ge 0$$
(7)

Where  $M_j$  denotes represented spectral characteristic of the component in concern, j and P denotes spectral characteristic of the observed pixel. Equation (7) can be rewritten by the following equation,

$$P = MA \tag{8}$$

where

$$egin{aligned} P &= [p_1, p_2, \cdots, p_n]^t \ M &= [m_1, m_2, \cdots, m_k] \ A &= [a_1, a_2, \cdots, a_k]^t \end{aligned}$$

If M is nonsingular matrix (square matrix), then mixing ratio vector can be estimated as follows,

$$A = M^{-1}P \tag{9}$$

Even if M is singular matrix (rectangle matrix), then mixing ratio vector can be expressed using the following Moore Penrose Inverse Matrix,  $M^+$ .

$$A = (M^{t}M)^{-1}M^{t}P = M^{+}P$$
(10)

It is possible to solve the equation with the following conditions,

$$|P-MA| o min \ u^t A = 1 \quad (u = [1,1,\cdots,1]^t) \ a_j \geq 0 \quad (j = 1,2,\cdots,k)$$

Introducing Lagrange multiplier, then the equation can be solved as follows,

$$F(A,\lambda) = \frac{1}{2}|P - MA|^2 - \lambda(u^t A - 1)$$
(12)

Thus mixing ratio vector A can be estimated through solving the following equations,

$$egin{aligned} &rac{\partial F}{\partial a_j} = -m_j^t(P-MA) - \lambda = 0 \quad (j=1,2,3) \ &rac{\partial F}{\partial \lambda} = -(u^tA-1) = 0 \end{aligned}$$

Equation (13) can be rewritten as follows, in the vector form,

$$-M^t(P - MA) - \lambda u = 0 \tag{14}$$

Thus

$$A = M^+ P + \lambda (M^t M)^{-1} u \tag{15}$$

Where

|M|

$$\lambda = \frac{1 - u^t M^+ P}{u^t (M^t M)^{-1} u} (M^t M)^{-1} u$$
(16)

Then A is rewritten from the equation (15) as follows,

$$A = M^{+}P + \frac{1 - u^{t}M^{+}P}{u^{t}(M^{t}M)^{-1}u}(M^{t}M)^{-1}u$$
(17)

Thus the mixing ratio vector can be estimated.

Although M denotes the representative observation vector, it contains error vector E as follows,

$$M = M_0 + E$$
$$P = M_0 A_0 \tag{18}$$

Namely, M is different from the true representative observation vector of  $M_0$ . Then the true mixing ratio is expressed as follows,

$$A_{1} = A_{0} - M^{+}EA_{0} + \frac{u^{t}M^{+}EA_{0}}{u^{t}(M^{t}M)^{-1}u}(M^{t}M)^{-1}u_{(19)}$$

Then estimation error of mixing ratio vector is expressed as follows,

$$\begin{aligned} |A_{1} - A_{0}| &= |-M^{+}EA_{0} + \frac{u^{t}M^{+}EA_{0}}{u^{t}(M^{t}M)^{-1}u}(M^{t}M)^{-1}u| \\ &\geq ||\frac{u^{t}M^{+}EA_{0}}{u^{t}(M^{t}M)^{-1}u}(M^{t}M)^{-1}u| - |M^{+}EA_{0}|| \\ &= |\frac{|\cos\beta|}{|\cos\alpha|} - 1||M^{+}EA_{0}| \end{aligned}$$

$$(20)$$

where  $\alpha$  denotes the angle between u and  $(M^t M)^{-1} u$ while  $\beta$  denotes the angle between u and  $M^{+} E A_0$ . Then estimation error can be expressed as follows,

$$A' = A_0 - M^+ E A_0$$
  
|A' - A\_0| = |M^+ E A\_0| (21)

Meanwhile the equation can be solved with the following conditions,

$$egin{aligned} & + & - & N ert 
ightarrow min \ & A &= & NP \ & u^t A &= 1 \end{aligned}$$

These conditions are corresponding to the following conditions,

$$|A-M^+P| 
ightarrow min$$
 $u^t A = 1$ 
(23)

Introducing Langrange multiplier, then

$$A = M^+ P + \lambda u \tag{24}$$

Minimizing the error,  $M^+$ -N, then

$$u^t (M^+ - V)P = 1 (25)$$

Introducing the following equation,

$$F(V,\lambda) = \frac{1}{2}|V|^2 - \lambda(u^t(M^+ - V)P - 1)$$
(26)

With the following conditional equations,

$$rac{\partial F}{\partial V_{ji}} = V_{ji} + \lambda p_i = 0$$
  $(i = 1, 2, \cdots, n), (j = 1, 2, \cdots, k)$   
 $rac{\partial F}{\partial \lambda} = -(u^t (M^+ - V)P - 1) = 0$ 
(27)

Then V can be rewritten as follows in the vector representation,

$$V = -\lambda u P^+ \tag{28}$$

and then Langrange multiplier can be expressed as follows,

$$\lambda = \frac{1 - u^t M^+ P}{|u|^2 |P|^2} \tag{29}$$

Thus all unknown variables are estimated as follows,

. . . . . .

$$V = -\frac{1 - u^{t}M^{+}P}{|u|^{2}|P|^{2}}uP^{t}$$

$$N = M^{+} + \frac{1 - u^{t}M^{+}P}{|u|^{2}|P|^{2}}uP^{t}$$

$$A = M^{+}P + \frac{1 - u^{t}M^{+}P}{|u|^{2}}u$$
(30)

Estimated mixing ratio  $A_2$  and estimation error,  $|A_2 - A_0|$  are expressed as follows,

$$A_{2} = A_{0} - M^{+}EA_{0} + \frac{1 - u^{t}M^{+}EA_{0}}{|u|^{2}}u$$
$$|A_{2} - A_{0}|^{2} = |M^{+}EA_{0}|^{2} - (\frac{1 - u^{t}M^{+}EA_{0}}{|u|})^{2}$$
(31)

*M* includes observation error so that *P* can be expressed with the following equation by considering the error  $\varepsilon$ ,

$$P = MA + \epsilon$$

$$P = [p_1, p_2, \cdots, p_n]^t$$

$$M = \begin{pmatrix} m_{11} & m_{12} & \cdots & m_{1k} \\ \vdots & \vdots & & \vdots \\ m_{n1} & m_{n2} & \cdots & m_{nk} \end{pmatrix}$$

$$A = [a_1, a_2, \cdots, a_k]^t$$

$$\epsilon = [\epsilon_1, \epsilon_2, \cdots, \epsilon_k]^t \qquad (32)$$

This method is referred to "Least Square Method".

If the components of M are followed by normal distribution as follows,

$$N(m_{ij}^*, \sigma_{ij}^2) \tag{33}$$

where

ο.

$$egin{aligned} m_i &= m_i^* \cdot A, m_i^* = [m_{i1}^*, m_{i2}^*, \cdots, m_{ik}^*,] \ \sigma_i^2 &= A^t \cdot S_i \cdot + \sigma_{ei}^2, S_i = diag(\sigma_{i1}^2, \sigma_{i2}^2, \cdots, \sigma_{ik}^2) \end{aligned}$$

Then the probability of observation vector M can be expressed as follows,

$$Q(p_i) = \frac{1}{(2\pi \cdot \sigma_i^2)^{\frac{1}{2}}} exp\left(-\frac{(p_i - m_i)^2}{2\sigma_i^2}\right)$$
(34)

It is rewritten in the vector form as follows,

$$Q(P) = \prod_{i=1}^{n} Q(p_i) \tag{35}$$

Best estimation of mixing ratio vector is to maximizing probability of observation vector so that,

$$R(P) = -ln(Q(P))$$
 $\Sigma_{j=1}^{k} A_{j} = 1, A_{j} \ge 0, (j = 1, \cdots, k)$ 
(36)

All of the meshed points of equation (36) can be calculated then the maximum probability of the observation vector can be found which results in estimation of mixing ratio. This method is referred to "Maximum Likelihood Method".

#### III. EXPERIMENTS

#### C. Data Used

ADEOS/AVNIR (Advanced Earth Observing Satellite/Advanced Visible to Near Infrared Radiometer) imagery data of Saroma Lake in Hokkaido, Japan which is acquired on February 1997 is used for the experiments. AVNIR consist of four spectral bands, blue (B1), green (B2), red (B3) and near infrared (B4) wavelength regions.

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From the original imagery data, 400 by 400 pixels of portion of image are extracted for the experiments. Instantaneous Field of View: IFOV is 16 meters. Figure 2 shows the averaged image among the four spectral bands of imagery data. The typical digital numbers of the spectral bands are shown in Table 1.

 
 TABLE I.
 TYPICAL PIXEL VALUE OF THE COMPONENTS FOR ALL SPECTRAL BANDS OF ADEOS/AVNIR

	B1	B2	B3	B4
Open Water	40.6875	26.5312	19.3125	13.5625
Thin Ice	119.141	107.609	101.078	89.1250
Thick Ice	130.000	118.609	113.984	106.188

Figure 2 shows the ADEOS/AVNIR image for experiments. Figure 3 (a), (b), and (c) shows open water, thin sea ice, and thick sea ice estimated by Maximum Likelihood Method.



Fig. 2. ADEOS/AVNIR (Advanced Earth Observing Satellite/Advanced Visible to Near Infrared Radiometer) imagery data of Saroma Lake in Hokkaido, Japan which is acquired on February 1997 is used for the experiments. AVNIR consist of four spectral bands, blue (B1), green (B2), red (B3) and near infrared (B4) wavelength regions. This image is averaged image among the four spectral bands

On the other hands, Figure 4 (a), (b), and (c) shows three components estimated by Least Square Method. Through comparisons between Figure 3 and 4, Maximum Likelihood Method derived open water, thin and thick sea ice is more likely in comparison to the visual perception.



(a)Open Water



(b)Thin Sea Ice



(c)Thick Sea Ice

Fig. 3. Estimated three components, open water, thin and thick ice derived from Maximum Likelihood Method



(a)Open Water



(b)Thin Sea Ice



(c)Thick Sea Ice

Fig. 4. Estimated three components, open water, thin and thick ice derived from Least Square Method

Figure 5 shows the extracted "Lead" which is defined as breaking portion of sea ice. The percentage ratio of open water ranged from 7 to 50% is defined as Lead. By using unmixing method, mixing ratio of open water can be estimated.

Therefore, the Lead is extracted. The extracted lead, however, is disconnected. Therefore, some consideration of connectivity of the piece of the disconnected lead is required.



Fig. 5. Estimated "Lead" of which mixing ratio of open water ranges from 7 to 50% derived from Maximum Likelihood Method

ADEOS/AVNIR Band 1 image is shown in Figure 6. By using band 1 of imagery data, connectivity between disconnected portions of lead can be found. This information is called as contextual information and can be extracted by using 3 by 3 or 5 by 5 pixel windows. For instance, connectivity between the pixel in concern and the 8 surrounding pixels can be checked with 3 by 3 windows. Using the connectivity, it is possible to connect between disconnected portions of lead. Figure 7 shows the connected lead using contextual information.



Fig. 6. ADEOS/AVNIR Band 1 image of Saroma, Hokkaido, Japan acquired on February 3 1997.



Fig. 7. Connected lead using contextual information.

#### IV. CONCLUSION

Unmixing method for estimation of mixing ratio of the components of which the pixel in concern consists based on inversion theory is proposed together with its application to sea ice estimation method with satellite based visible to near infrared radiometer data. Through comparative study on the different unmixing methods with remote sensing satellite imagery data, it is found that the proposed inversion theory based unmixing method is superior to the other methods. Also it is found that the proposed unmixing method is applicable to sea ice concentration estimations. It is also found that contextual information is effective to connect disconnected portion of lead.

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