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Accurate Topological Measures for Rough Sets

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***Abstract*—Data granulation is considered a good tool of decision making in various types of real life applications. The basic ideas of data granulation have appeared in many fields, such as interval analysis, quantization, rough set theory, Dempster-Shafer theory of belief functions, divide and conquer, cluster analysis, machine learning, databases, information retrieval, and many others. Some new topological tools for data granulation using rough set approximations are initiated. Moreover, some topological measures of data granulation in topological information systems are defined. Topological generalizations using $\delta\beta$ -open sets and their applications of information granulation are developed.**

Keywords—component; Knowledge Granulation; Topological Spaces; Rough Sets; Rough Approximations; Data Mining; Decision Making

I. INTRODUCTION

Granulation of the universe involves the decomposition of the universe into parts. In other words, the grouping individual elements or objects into classes, based on offering information and knowledge [7, 14,15, 21, 36,37, 42-45]. Elements in a granule are pinched together by indiscernibility, similarity, proximity or functionality [43]. The starting point of the theory of rough sets is the indiscernibility of objects or elements in a universe of concern [14,15, 17-20, 51,52, 21-22].

The original rough set theory was based on an equivalent relation on a finite universe U. For practical use, there have been some extensions on it. One extension is to replace the equivalent relation by an arbitrary binary relation; the other direction is to study rough set via topological method [8, 14]. In this work, we construct topology for a family covering rough sets.

In [40] addressed four operators on a knowledge base, which are sufficient for generating new knowledge structures. Also, they addressed an axiomatic definition of knowledge granulation in knowledge bases.

Rough set theory, proposed by Pawlak in the early 1980s [18, 51-52], is an expansion of set theory for the study of intelligent systems characterized by inexact, uncertain or insufficient information. Moreover, this theory may serve as a new mathematical tool to soft computing besides fuzzy set theory [42-45] and has been successfully applied in machine learning, information sciences, expert systems, data reduction, and so on [28-33,34, 1-13]. In recent times, lots of researchers are interested to generalize this theory in many fields of applications [1-10].

In Pawlak's novel rough set theory, partition or equivalence (indiscernibility) relation is an important and primeval concept. But, partition or equivalence relation is still limiting for many applications. To study this matter, several interesting and having an important effect generalization to equivalence relation have been proposed in the past, such as tolerance relations, similarity relations [51], topological bases and subbases [52, 2,6] and others [4,5,11]. Particularly, some researchers have used coverings of the universe of discourse for establishing the generalized rough sets by coverings [11-14]. Others [24-26,27-33] combined fuzzy sets with rough sets in a successful way by defining rough fuzzy sets and fuzzy rough sets. Furthermore, another group has characterized a measure of the roughness of a fuzzy set making use of the concept of rough fuzzy sets [34-38]. They also suggested some possible real world applications of these measures in pattern recognition and image analysis problems [24,41-46].

Topological notions like semi-open, pre-open, $\beta-$open sets are as basic to mathematicians of today as sets and functions were to those of last century [48-52]. Then, we think the topological structure will be so important base for knowledge extraction and processing.

The topology induced by binary relations on the universes of information systems is used to generalize the basic rough set concepts. The suggested topological operations and structure open up the way for applying affluent more of topological facts and methods in the process of granular computing. In particular, the notion of topological membership function is introduced that integrates the concept of rough and fuzzy sets [17-20].

In this paper, we indicated some topological tools for data granulation by using new topological tools for rough set approximations. Moreover, we introduced using general binary relations a refinement data granulation instead of the classical equivalence relations. Section 1 gives a brief overview of data granulation structures in the universe using equivalence and general relations. Fundamentals of rough set theory under general binary relations are the main purpose of Section 2. Section 3 studies the topological data granulation properties of topological information systems. Explanation of topological data granulation in information systems appears in

Section 4. In Section 5 we are given some more accurate topological tools for data granulation using $\delta\beta-$ open sets approach. The conclusions of our work are presented in Section 6.

II. Essentials of Rough Set Approximations Under General Binary Relations

In rough set theory, it is usually assumed that the knowledge about objects is restricted by some indiscernibility relations. The Indiscernibility relation is an equivalence relation which is interpreted so that two objects are equivalent if we can't distinguish them using our information. This means that the objects of the given universe U indiscernible by R into three classes with respect to any subset $X \subseteq U$:

Class 1: the objects which surely belong to X ,

Class 2: the objects which possibly belong to X ,

Class 3: the objects which surely not belong to X ,

The object in Class 1 form the lower approximation of X , and the objects of Class 1 and 3 form together its upper approximation. The boundary of X consists of objects in Class 3. Some subsets of U are identical to both of them approximations and they are called crisp or exact; otherwise, the set is called rough.

For any approximation space $A=(U,R)$, where R is an equivalence relation, lower and upper approximations of a subset $X \subseteq U$, namely $\underline{R}(X)$ and $\overline{R}(X)$ are defined as follows:

$$\underline{R}(X)=\{x\in U:[x]_R \subset X\},$$
$$\overline{R}(X)=\{x \in U:[x]_R \cap X \neq \phi\}.$$

The lower and upper approximations have the following properties:

For every $X,Y \subset U$ from the approximation space $A=(U,R)$ we have:

1. $\underline{R}(X) \subseteq X \subseteq \overline{R}(X)$,
2. $\underline{R}(U)=\overline{R}(U)=U$,
3. $\underline{R}(\phi)=\overline{R}(\phi)=\phi$,
4. $\overline{R}(X \cup Y)=\overline{R}(X) \cup \overline{R}(Y)$,
5. $\underline{R}(X \cup Y) \supseteq \underline{R}(X) \cup \underline{R}(Y)$,
6. $\overline{R}(X \cap Y) \subseteq \overline{R}(X) \cap \overline{R}(Y)$,
7. $\underline{R}(X \cap Y)= \underline{R}(X) \cap \underline{R}(Y)$,
8. $\overline{R}(-X)=-\underline{R}(X)$,
9. $\underline{R}(-X)=-\overline{R}(X)$,
10. $\overline{R}(\overline{R}(X))=\underline{R}(\overline{R}(X))=\overline{R}(X)$,
11. $\underline{R}(\underline{R}(X))=\overline{R}(\underline{R}(X))=\underline{R}(X)$,
12. *If* $X \subseteq Y$ *, then* $\overline{R}(X) \subseteq \overline{R}(Y)$ *and* $\underline{R}(X) \subseteq \underline{R}(Y)$.

The equality in all properties happens when $\underline{R}(X)=\overline{R}(X)=X$. The proof of all these properties can be found in [17-23,51].

Furthermore, for a subset $X \subseteq U$, a rough membership function is defined as follows: $\mu_X(x)=\dfrac{|[x]_R \cap X|}{|[x]_R|}$, where $|X|$ denotes the cardinality of the set X . The rough membership value $\mu_X(x)$ may be interpreted as the conditional probability that an arbitrary element belongs to X given that the element belongs to $[x]_R$.

Based on the lower and upper approximations, the universe U can be divided into three disjoint regions, the positive $POS(X)$, the negative $NEG(X)$ and the boundary $BND(X)$,where:

$$POS(X)=\underline{R}(X)$$
$$NEG(X)=U-\overline{R}(X)$$
$$BND(X)=\overline{R}(X)-\underline{R}(X)$$

Considering general binary relations in [18,52] is an extension to the classical lower and upper approximations of any subset X of U . $\beta=\{R_x : x \in X\}$ is the base generated by the general relation defined in [17,52]. The general forms based on β are defined as follows:

$$\underline{R}_\beta(X)=\bigcup\{B:B\in\beta_x,\, B\subset X\},$$
$$\overline{R}_\beta(X)=\bigcup\{B:B\in\beta_x,\, B\cap X \neq \phi\},$$ where $\beta_x=\{B\in\beta : x\in B\}$.

For data granulation by any binary relation, in [E. Lashein (2005)] a rough membership function is defined as follows:

$$\mu_X(x)=\frac{|X \cap (\bigcap\beta_x)|}{|\bigcap\beta_x|}.$$

III. Rough Sets of Equivalence and General Binary Relations

Indiscernibility as defined by equivalence relation represents a very restricted type of relationships between elements and universes. The procedure to granule the universe by general binary relations is introduced in [6].

A topological space [1,2] is a pair (X,τ) consisting of a set X and a family τ of subset of X satisfying the following conditions:

(1) $\phi, X \in \tau$,

(2) τ is closed under arbitrary union,

(3) τ is closed under finite intersection.

The pair (X,τ) is called a topological space. The elements of X are called points . The subsets of X belonging to τ are called open sets. The complement of the open subsets are called closed sets. The family τ of all open subsets of X is also called a topology for X . $cl(A)=\bigcap\{F \subseteq X : A \subseteq F \text{ and } F \text{ is closed}\}$ is called τ -closure of a subset $A \subset X$.

Obviously, $cl(A)$ is the smallest closed subset of X which contains A. Note that A is closed iff $A = cl(A)$. $int(A)=\bigcup\{G \subseteq X : G \subseteq A \text{ and } G \text{ is open}\}$ is called the τ -interior of a subset $A \subseteq X$. Manifestly, $int(A)$ is the union of all open subsets of X which contained in A . Make a note of that A is open iff $A = int(A)$. $b(A) = cl(A) - int(A)$ is called the τ -boundary of a subset $A \subseteq X$.

For any subset A of the topological space (X,τ) , $cl(A)$, $int(A)$ and $b(A)$ are closure, interior, and boundary of A respectively. The subset A is exact if $b(A)=\phi$, otherwise A is rough. It is clear that A is exact iff $cl(A) = int(A)$. In Pawlak space a subset $A \subseteq X$ has two possibilities either rough or exact.

In later years a number of generalizations of open sets have been considered [21-23]. We talk about some of these generalizations concepts in the following definitions.

Let U be a finite universe set and R is any binary relation defined on U , and $rR(x)$ be the set of all elements which are in relation to certain elements x in U from right for all $x \in U$, in symbols $rR(x) = \{xR, x \in U\}$ where $xR = \{y : (x, y) \in R; x, y \in U\}$.

Let β be the general knowledge base (topological base) using all possible intersections of the members of $rR(x)$. The component that will be equal to any union of some members of β must be misplaced.

IV. Topological Generalizations of Rough Sets

Let $A = (U, R)$ be an approximation space where R is any binary relation defined on U . Then we can define two new approximations as follows:

$$\underline{\tau}_\beta(X) = X \cap \underline{R}_\beta(\overline{R}_\beta(X)),$$
$$\overline{\tau}_\beta(X) = X \cup \overline{R}_\beta(\underline{R}_\beta(X)).$$

The topological lower and the topological upper approximations have the following properties:

For every $X, Y \subset U$ and every approximation space $A = (U, R)$ we have:

1. $\underline{\tau}_\beta(X) \subseteq X \subseteq \overline{\tau}_\beta(X)$,
2. $\underline{\tau}_\beta(U) = U = \overline{\tau}_\beta(U)$,
3. $\overline{\tau}_\beta(\phi) = \underline{\tau}_\beta(\phi) = \phi$,
4. $\overline{\tau}_\beta(X \cup Y) \supset \overline{\tau}_\beta(X) \cup \overline{\tau}_\beta(Y)$,
5. $\underline{\tau}_\beta(X \cup Y) \supset \underline{\tau}_\beta(X) \cup \underline{\tau}_\beta(Y)$,
6. $\overline{\tau}_\beta(X \cap Y) \subset \overline{\tau}_\beta(X) \cap \overline{\tau}_\beta(Y)$,
7. $\underline{\tau}_\beta(X \cap Y) \subseteq \underline{\tau}_\beta(X) \cap \underline{\tau}_\beta(Y)$,
8. $\overline{\tau}_\beta(-X) = -\overline{\tau}_\beta(X)$,
9. $\underline{\tau}_\beta(-X) = -\underline{\tau}_\beta(X)$,
10. $\overline{\tau}_\beta(\overline{\tau}_\beta(X)) = \overline{\tau}_\beta(X)$,
11. $\underline{\tau}_\beta(\underline{\tau}_\beta(X)) = \underline{\tau}_\beta(X)$,
12. $If\ X \subseteq Y,\ then\ \overline{\tau}_\beta(X) \subseteq \overline{\tau}_\beta(Y)\ and\ \underline{\tau}_\beta(X) \subseteq \underline{\tau}_\beta(Y).$

Given that topological lower and topological upper approximations satisfy that: $\underline{R}_\beta(X) \subseteq \underline{\tau}_\beta(X) \subseteq X \subseteq \overline{\tau}_\beta(X) \subseteq \overline{R}_\beta(X) \subseteq U$ this enables us to divide the universe U into five disjoint regions (granules) as follows: (See Figure 1)

1. $POS_\beta(X) = \underline{R}_\beta(X)$,
2. $\tau - POS(X) = \underline{\tau}_\beta(X) - \underline{R}_\beta(X)$,
3. $\tau - BND(X) = \overline{\tau}_\beta(X) - \underline{\tau}_\beta(X)$,
4. $\tau - NEG(X) = \overline{R}_\beta(X) - \overline{\tau}_\beta(X)$,
5. $NEG_\beta(X) = U - \overline{R}_\beta(X)$.

The following theorems study the properties and relationships among the above regions namely boundary, positive and negative regions.

Theorem 4.1 let $IS = (U, A, \tau_R)$ be a topological information system and for any subset $X \subset U$ we have:

(1) $\tau - BND(X) \cap \underline{\tau}_\beta(X) = \phi$,

(2) $\tau - BND(X) \cap \tau - NEG(X) = \phi$,

(3) $\overline{\tau}_\beta(X) = \underline{\tau}_\beta(X) \cup \tau - BND(X)$,

(4) $\underline{\tau}_\beta(X), \tau - NEG(X)$ and $\tau - BND(X)$ are disjoint granules of U.

Proof: You can make use of Figure 1.

Theorem 4.2 let $IS = (U, A, \tau_R)$ be a topological information system and for any subsets $X, Y \subset U$ we have:

(1) $\tau - BND(U) = \phi$,

(2) $\tau - BND(X) = \tau - BND(U - X)$,

(3) $\tau - BND(\tau - BND(X)) \subset \tau - BND(X)$,

$\tau - BND(X \cap Y) \subset \tau - BND(X) \cup \tau - BND(Y)$

Proof: (1) and (2) is obvious, by definitions.

(3)

$$\tau - BND(\tau - BND(X))$$
$$= \tau - BND(\overline{\tau}_\beta(X) \cap \overline{\tau}_\beta(U - X))$$
$$= \overline{\tau}_\beta(\overline{\tau}_\beta(X) \cap \overline{\tau}_\beta(U - X))$$
$$\cap \overline{\tau}_\beta(U - (\overline{\tau}_\beta(X) \cap \overline{\tau}_\beta(U - X)))$$
$$\subset \overline{\tau}_\beta(X) \cap \overline{\tau}_\beta(U - X) = \tau - BND(X).$$

(4)

$$\tau - BND(X \cap Y) = \overline{\tau}_\beta(X \cap Y) \cap \overline{\tau}_\beta(U - X \cap Y)$$

Theorem 4.3 let $IS = (U, A, \tau_R)$ be a topological information system and for any subset $X, Y \subset U$ we have:

(1) $U = \tau - NEG(\phi)$,

(2) $\tau - NEG(X) = \underline{\tau}_\beta(U - X)$,

(3) $X \cap \tau - NEG(X) = \phi$,

(4) $\tau - NEG(U - \tau - NEG(X)) = \tau - NEG(X)$,

(5) $\tau - NEG(X \cup Y) \subset \tau - NEG(X) \cup \tau - NEG(Y)$,

(6) $\tau - NEG(X \cap Y) \supset \tau - NEG(X) \cap \tau - NEG(Y)$

Proof: (1), (2), (3) and (4) are obvious.

(5)

$$\tau - NEG(X \cup Y)$$
$$= U - \overline{\tau}_\beta(X \cup Y) \subset U - (\overline{\tau}_\beta(X) \cup \overline{\tau}_\beta(Y))$$
$$= (U - \overline{\tau}_\beta(X)) \cap (U - \overline{\tau}_\beta(Y))$$
$$\subset \tau - NEG(X) \cup \tau - NEG(Y)$$

(6)

$$\tau - NEG(X) \cap \tau - NEG(Y)$$
$$= (U - \overline{\tau}_\beta(X)) \cap (U - \overline{\tau}_\beta(Y))$$
$$= U - (\overline{\tau}_\beta(X) \cup \overline{\tau}_\beta(Y)) \subset U - \overline{\tau}_\beta(X \cap Y)$$
$$= \tau - NEG(X \cap Y)$$

Example 4.1 let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$ be the universe of 7 patients have data sheets shown in Table I with possible dengue symptoms. If some experts give us the general relation R defined among those patients as follows:

TABLE I. PATIENTS INFORMATION SYSTEM

U	Conditional Attributes (C)			Decision (D)
	Temperature	Flu	Headache	Dengue
u1	Normal	No	No	No
u2	High	No	No	No
u3	Very High	No	No	Yes
u4	High	No	Yes	Yes
u5	Very High	No	Yes	Yes
u6	High	Yes	Yes	Yes
u7	Very High	Yes	Yes	Yes

$$R = \{(u_1,u_1),(u_1,u_7),(u_2,u_2),(u_3,u_3),$$
$$(u_3,u_6),(u_4,u_4),(u_5,u_5),(u_6,u_6)$$
$$,(u_7,u_7)\}.$$

The topological knowledge base will take the following form:

$$\beta = \{\{u_1,u_7\},\{u_2\},\{u_3,u_6\},\{u_4\},\{u_5\},\{u_6\},\{u_7\}\}$$

For some patients $X = \{u_2, u_3, u_7\}$ the upper and lower approximations based on the topological knowledge base are given by:

$\overline{R}_\beta(X) = \{u_1, u_2, u_3, u_6, u_7\}$, and $\underline{R}_\beta = \{u_2, u_7\}$.

By using the lower and upper approximations, the granules of universe are three disjoint regions as follows:

$$POS_\beta(X) = \underline{R}_\beta(X) = \{u_2, u_7\},$$
$$BND_\beta(X) = \overline{R}_\beta(X) - \underline{R}_\beta(X) = \{u_1, u_3, u_6\},$$
$$NEG_\beta(X) = U - \overline{R}_\beta(X) = \{u_4, u_5\}.$$

According to the topological knowledge base we can easily see that:

$\overline{\tau}_\beta(X) = \{u_1, u_2, u_3, u_7\}$, $\underline{\tau}_\beta(X) = \{u_2, u_3, u_7\}$.

Then we have the following granules of the universe:

1. $POS_\beta(X) = \{u2, u7\}$,

2. $\tau - POS(X) = \{u3\}$,
3. $\tau - BND(X) = \{u1\}$,
4. $\tau - NEG(X) = \{u6\}$,
5. $NEG_{\beta}(X) = \{u4, u5\}$.

V. New Topological Generalizations of Rough Sets

In this section, we used the topological tool $\delta\beta$ -open sets to introduce the concepts of $\delta\beta$ -lower and $\delta\beta$ -upper approximations. The suggested model helps in decreasing the boundary region of concepts in information systems. Also, we use the topological measure $\alpha_{R_{\delta\beta}}$ is used as a topological accurate measure of data granulation correctness.

For any subset X of a topological space (U,τ). The δ -closure of a subset X is defined by $cl_{\delta}(X) = \{x \in U : X \cap int(cl(G)) \neq \phi, G \in \tau \text{ and } x \in G\}$. A set X is called δ -closed if $X = cl_{\delta}(X)$. The complement of a δ -closed set is called δ -open.

Notice that $int_{\delta}(X) = U \setminus cl_{\delta}(U \setminus X)$.

A subset X of a topological space (U,τ) is called $\delta\beta$ -open if $X \subseteq cl(int(cl_{\delta}(X)))$.

Let (U,τ) be a topological space and $X \subseteq U$, the following new topological tools of any subset X are defined as follows [1,2,6]:

- Regular open tool if $X = Int(Cl(X))$.
- Semi-open tool if $X \subset Cl(Int(X))$.
- $\alpha -$ open tool if $X \subset Int(Cl(Int(X)))$.
- Pre-open tool if $X \subset Int(Cl(X))$.
- Semi pre open tool ($\beta -$ open) if $X \subset Cl(Int(Cl(X)))$.

The family of all $\delta\beta$ -open sets of U is denoted by $\delta\beta O(U)$. The complement of $\delta\beta$ -open set is called $\delta\beta$ -closed set. The family of $\delta\beta$ -closed sets are denoted by $\delta\beta C(U)$.

Let X be a subset of a topological space (U,τ), then we have:

(i) The union of all $\delta\beta$ -open sets contained inside X is called the $\delta\beta$ -interior of X and is denoted by $\beta int_{\delta}(X)$.

(ii) The intersection of all $\delta\beta$ -closed sets containing X is called the $\delta\beta$ -closure of X and is denoted by $\beta cl_{\delta}(X)$.

Lemma 6.1 For a subset X of a topological space (U,τ) we have:

(i) $\beta int_{\delta}(X) = X \cap cl(int(cl(X)))$.

(ii) $\beta cl_{\delta}(X) = X \cup int(cl(int(X)))$.

$\delta\beta$ -open sets is stronger than any topological near open sets such as δ -open, regular open, semi-open, $\alpha -$ open, pre-open, β -open.

The following example illustrates the above note.

Example 5.1 Let (U,τ) be a topological space where, $U = \{a,b,c,d,e\}$ and $\tau = \{U, \varphi, \{d\}, \{e\}, \{a,d\}, \{d,e\}, \{a,d,e\}, \{b,c,e\}, \{b,c,d,e\}\}$. We have $\{a,c\} \in \delta\beta O(U)$ but $\{a,c\} \notin \delta O(U)$, $\{b,d,e\} \in \delta\beta O(U)$ but $\{b,d,e\} \notin RO(U)$, $\{a,e\} \in \delta\beta O(U)$ but $\{a,e\} \notin PO(U)$, $\{c\} \in \delta\beta O(U)$ but $\{c\} \notin \beta O(U)$, $\{b\} \in \delta\beta O(U)$ but $\{b\} \notin SO(U)$ and $\{c,d\} \in \delta\beta O(U)$ but $\{c,d\} \notin \alpha O(U)$. Where $\delta O(U)$, $RO(U)$, $SO(U)$, $\alpha O(U)$, $PO(U)$ and $\beta O(U)$ denoted the family of all δ -open, regular open, semi-open, $\alpha -$ open, pre-open and β -open sets of U respectively.

Arbitrary union of $\delta\beta$ -open sets is again $\delta\beta$ -open set, but the intersection of two $\delta\beta$ -open sets may not be $\delta\beta$ -open set. Thus the $\delta\beta$ -open sets in a space U do not form a topology.

Let U be a finite non-empty universe. The pair $(U, R_{\delta\beta})$ is called a $\delta\beta$ -approximation space where $R_{\delta\beta}$ is a general relation used to get a subbase for a topology τ on U which generates the class $\delta\beta O(U)$ of all $\delta\beta$ -open sets.

Example 6.2 Let $U = \{a,b,c,d,e\}$ be a universe and a relation R defined by $R = \{(a,a), (a,e), (b,c), (b,d), (c,e), (d,a), (d,e), (e,e)\}$, thus $aR = dR = \{a,e\}$,

$bR=\{c,d\}$ and $cR=eR=\{e\}$. Then the topology associated with this relation is $\tau=\{U, \phi, \{e\}, \{a,e\}, \{c,d\}, \{c,d,e\}, \{a,c,d,e\}\}$ earned $\delta\beta O(U)=P(U)-\{b\}$. So $(U,R_{\delta\beta})$ is a $\delta\beta$ -approximation space.

Let $(U,R_{\delta\beta})$ be a $\delta\beta$ - approximation space. $\delta\beta$ -lower approximation and $\delta\beta$ -upper approximation of any non-empty subset X of U is defined as:

$$\underline{R}_{\delta\beta}(X)=\bigcup\{G\in\delta\beta O(U): G\subseteq X\},$$

$$\overline{R}_{\delta\beta}(X)=\bigcap\{F\in\delta\beta C(U): F\supseteq X\}.$$

We see that: $$\underline{R}(X)\subseteq\underline{R}_{\beta}(X)\subseteq\underline{R}_{\delta\beta}(X)\subseteq X \subseteq\overline{R}_{\delta\beta}(X)\subseteq\overline{R}_{\beta}(X)\subseteq\overline{R}(X).$$

Let $(U,R_{\delta\beta})$ be a $\delta\beta$ - approximation space, $X\subseteq U$. From the relation $$int(X)\subseteq\beta int(X)\subseteq\delta\beta int(X)\subseteq X \subseteq\delta\beta cl(X)\subseteq\beta cl(X)\subseteq cl(X),$$ The Universe U can be separated into divergent 24 granules with respect to any $X\subseteq U$.

We can distinguish the degree of completeness of granules of U by the topological tool named $\delta\beta$ -accuracy measure defined for any granule $X\subseteq U$ as follows:

$$\alpha_{R_{\delta\beta}}(X)=\frac{|\underline{R}_{\delta\beta}(X)|}{|\overline{R}_{\delta\beta}(X)|} \quad where \quad X\neq\phi.$$

Example 5.2 According to Example 5.1 we can construct the following table (Table II) showing the degree of accuracy measure $\alpha_R(X)$, β -accuracy measure $\alpha_{R_\beta}(X)$ and $\delta\beta$ -accuracy measure $\alpha_{R_{\delta\beta}}(X)$ for some granules of U.

TABLE II. ACCURACY MEASURES OF SOME GRANULES

Some granules	Pawlak's accuracy	β -accuracy	$\delta\beta$ -accuracy
{b, d}	0%	100%	100%
{b, e}	33.3%	66.6%	100%
{a, b, e}	66.6%	100%	100%
{a, c, d}	50%	66.6%	100%
{b, c, d, e}	60%	80%	100%

We see that the degree of accuracy of the granule $\{b,c,d,e\}$ using Pawlak's accuracy measure equal to 60%, using β -accuracy measure equal to 80% and using $\delta\beta$ -accuracy measure equal to 100%. Accordingly $\delta\beta$ -accuracy measure is more precise than Pawlak's accuracy and β -accuracy measures.

VI. CONCLUSIONS AND APPLICATION NOTES

In the near future is the completion of a new paper for the application of the granules concepts of this paper in medicine especially in the field of heart disease in collaboration with specialists in this field. We designed a JAVA application program novelty to generate granules division automatically once you select points covered by the heart scan and the medical relationship among them using topology defined on it. The program works under any operating system but needs to be a great RAM memory and strong processor to end the division of the millions of points to the granules in seconds.

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