On Some Methods for Dimensionality Reduction of ECG Signals

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***Abstract*—Dimensionality reduction with two methods, namely, Laplacian Eigenmap (LE) and Locality Preserving Projections (LPP) is studied for normal and pathological noisy and noiseless ECG patterns. Besides, the possibility of using compressed sensing (CS) as a method of dimensionality reduction is also analyzed. The classification rate for the initial domain as well as in manifolds of various dimensions for the three cases are presented and compared.**

***Keywords*—Dimensionality reduction; compressed sensing; electrocardiography (ECG)**

I. Introduction

In the last years, dimensionality reduction methods have been widely investigated. The main idea about these methods is to represent high-dimensional raw data on intrinsic lower dimensional spaces. The main targets are either to reduce the computation costs for the raw data or to represent the data in a friendlier manner.

Wireless biomedical sensors are easy to use for long-term monitoring, especially outside the hospital, offering the possibility of better results able to substantially improve the patient's health and quality of life. Since electrocardiography (ECG) signals recorded from the electrical activity of the heart over a period of time have been used for diagnosis in many diseases, ECG tele-monitoring is accepted as an encouraging method in tele-medicine. Considering the importance of ECG signal recording, we are confronted with the problem of storing, transmitting and processing/classification the signals preferably in real time. One of the solutions to all these problems is the application of the dimensionality reduction of data.

Several reasons for using dimensionality reduction are [1]:

- The space required for storing data is reduced as the number of dimensions decreases.
- Reduced dimensions lead to less computing / training time.
- Some algorithms do not work well for large size data. Thus, the diminution of these dimensions must be in order for the algorithm to be useful.
- It considers multi-collinearity by removing redundant features.
- In numerous cases, for datasets with high dimensional not all measured variables are "important" for comprehension the basic phenomena of interest.
- Helps view data. It is difficult or impossible to view data in dimensions larger than 3D, therefore, reducing the space dimension to 2D or 3D, permits representing the data and thus plot and observe more clearly the models that appear and/or even better understand their spatial representation.

In this paper we will use three methods for dimensionality reduction of ECG signals, namely, Laplacian Eigenmaps (LE) and Locality Preserving Projections (LPP), as well as a third method, the compressed acquisition (compressed sensing – CS). For testing these methods, we used ECG signal segments belonging to 8 distinct classes.

LPP is a typical graph-based dimensionality reduction method, consisting of projective maps based on solving a variational problem that optimally conserve the neighborhood structure of the data set. When the high dimensional data lies on a low dimensional manifold embedded in the ambient space, the LPP are obtained by finding the optimal linear approximations to the eigen functions of the Laplace Beltrami operator on the manifold [2,3].

The main advantage of LE is that it reduces dimensionality by keeping the local and global structure even when the data is on a manifold [4,5].

To increase precision in classifications while reducing dimensionality all ECG segments have been processed to have the R wave centered. In order to conserve classes on the manifold, the main idea is to preserve neighbors as unchanged as possible. For all cases of dimensionality reduction with the three tested algorithms, the qualitative/quantitative evaluation is the classification rate. Therefore, classification rate obtained with the original ECG patterns with the classification rates obtained on new ECG patterns on which dimensionality reductions with Laplacian Eigenmaps, Locality Preserving Projections and compressed sensed were applied will be compared.

II. Background

A. Dimensionality Reduction Techniques

In the linear case the dimensionality problem can be stated as follows: Starting from a dataset of n vectors:

$X = [x_1 x_2 \dots x_n],\ x_i \in R^N$

find, a projection matrix $P \in R_{N,M}$ which leads to the low dimensional dataset

$Y = [y_1 y_2 \dots y_n],\ y_i \in R^M$

M<N through the linear projection

$Y = P^T X$

Depending on the specific target represented, P can be learned through various dimension reduction techniques. In our case the above approach corresponds to LPP and CS. The LE is closely related to the above techniques except for the non-linear character, both techniques conserving the local manifold structure [1].

a) Locality Preserving Projection: LPP is a linear projective map that arises by solving a variational problem which optimally preserves the neighborhood structure of the data set.

The LPP algorithm is proposed for the purpose of deriving a linear subspace with manifold data structure. The LPP algorithm can be viewed as linear approximate version derived by Laplacian Eigenmap algorithm.

The first step in LLP is the construction of an adjacency graph G with n nodes, the number of testing signals, each node corresponding to a signal in the dataset; the ith node corresponds to the data x_i. We put an edge between any x_i and all points within a K-neighbourhood [2-3]. Based on the distance between nodes choose the weights w_{ij} with the formula.

$w_{ij} = e^{-\|x_i - x_j\|^2/\sigma}$

where, σ is a positive constant. The similarity matrix w_{ij} of the graph G models the local structure of the data manifold in the original space. The last step is to solve the generalized eigen-decomposition problem:

$XLX^T\mathbf{a} = \Lambda XDX^T\mathbf{a}$,

Where:

D = Diagonal matrix

L = Laplacian matrix

Λ = a diagonal matrix with eigenvalue in ascending order

a = the corresponding eigenvector matrix.

To finish, P matrix is constructed with the first M columns in **a**. Note that the projected data to the subspace from different classes can be separated in general with nonlinear decision boundary [2-3].

b) Laplacian Eigenmap: The first two stages of the LE method are the same as those of LLP while the last stage consists of computing the eigenvectors and eigenvalues for the generalized eigenvector problem [4, 5]:

$L\mathbf{f} = \lambda D\mathbf{f}$

where, $D = (d_{ij})$ is an $(n \times n)$ diagonal matrix with,

$d_{ii} = \sum_{j \in N_i} w_{ij}$

L = D−W being the Laplacian matrix that can be thought as an operator on functions defined on the vertices of G.

At finish, we eliminate the eigenvector f_0 corresponding to the 0 eigenvalue and utilize the next m eigenvectors corresponding to the next m eigenvalues in increasing order for embedding in an m dimensional Euclidean space:

$x_i \rightarrow (f_1(i), \dots, f_m(i))$

where, $f_0, \dots, f_{k-1}$ are the solutions of generalized eigenvector problem [1,4,5].

B. Compressive Sensing

CS is a method that can be used to acquire signals with smaller number of measurements than the Nyquist rate in order to approximate sparse signals. According to CS theory a signal x can be characterized using the projections:

$y = \emptyset x$

where, $x \in R^N$, $y \in R^M$ is the measurement vector and $\emptyset \in R_{M,N}$ is the CS measurement or projection matrix whose entries are independent identically distributed (i.i.d) samples [6,7]. According to CS theory, the signal x can be approximately reconstructed from its projection by using an appropriate dictionary. However, in our approach we do not aim at signal reconstruction but only use the low dimensional projection vector y for classifications [6,7].

III. Experimental Results

To order to study the possibilities of dimension reduction using LE, LPP and CS method, we have considered 44 ECG signals from the MIT-BIH Arrhythmia databank acquired at a sampling frequency of 360Hz, with 11 bits / sample [8]. The databank also contains annotation files with the index of the R wave and the class to which each ECG pattern belongs.

According to the annotations files, eight major classes have been recognized, i.e., seven classes of pathological classes: atrial premature beat (A), left bundle branch block beat (L), right bundle branch block beat (R), premature ventricular contraction (V), fusion of ventricular and normal beat (F), paced beat (/), fusion of paced and normal beat (f) and a class of normal beats (N).

We used the same segmentation as in [9] to increase classification rate. So, a cardiac pattern begins in the middle of the RR interval and ends in the middle of the next RR interval. In the cardiac pattern thus obtained the R wave will be placed in the middle by resampling the waveforms on both sides of R. In this way patterns with the cardiac R wave centered have been obtained. So, all cardiac patterns are of dimension 301, with the R wave placed on the 150-th sample.

We constructed a database, namely a data set with 5608 patterns, having 701 patterns for each of the above classes. All patterns were normalized to unity norms. However, sensitivity to normalization was observed only in the case of LPP.

To classification in the initial 301 dimensional signal space we have used the KNN classifier using the Euclidean distance and the membership decision was based on the nearest neighbor.

For the original normalized ECG data the classification rate for the eight classes analyzed has been found to be 94.92% [11].

Fig. 1 shows the results for all tested methods for dimensionality reduction. It can be seen that for LE and CS the classification rate is increasing and once a maximum is reached, the classification rate stabilizes around that value. For very small values of the space dimension the best results are obtained with LE. Thus, for space dimension equal to 2 a classification rate of 82.61% is obtained, while for space dimension 8 the results are comparable for all three tested methods. If we refer to the maximum values achieved in terms of classification, then LPP offers the best results, i.e., for space dimension equal to 25, a 94.37% classification rate was obtained [10]. Several results regarding dimensionality reduction for the three tested methods are presented in Table I.

Since many techniques are noise-sensitive, we tested all three algorithms with waveforms with 8% added noise normally distributed.

Fig. 2 shows the classification rate obtained by LPP for ECG segments with and without noise. It is found that the locality-preserving character of the LPP method makes it relatively insensitive to noise because the classification rate varies significantly in the presence of 8% noise.

Fig. 3 shows the classification rate obtained by Laplacian Eigenmaps for ECG segments with and without noise. There are some small differences, but they are not significant so that LE can be considered almost insensitive to the presence of noise.

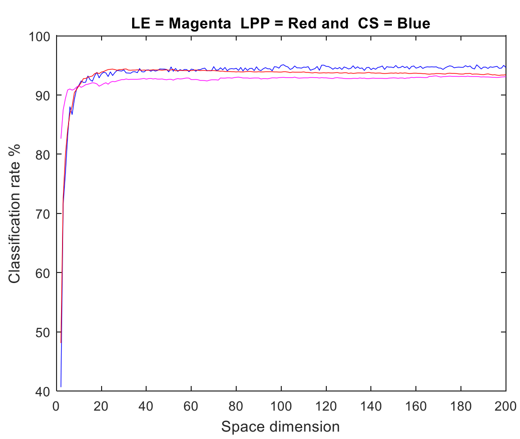

Fig. 1. Classification Rate vs. Space Dimension with LPP, LE and CS (Sigma = 5, Neighborhood k = 9) for Noiseless and Normalised ECG Patterns.

TABLE. I. CLASSIFICATION RATE % VS. SPACE DIMENSION FOR LLP, LE AND CS (SIGMA = 5, NEIGHBORHOOD K=9)

Space dimension	LE	LPP	CS
2	82,61	48,10	40,64
3	87,46	72,55	71,47
4	89,40	79,87	76,79
5	90,85	84,14	83,06
6	91,01	86,83	87,99
7	90,82	88,11	86,72
8	90,98	90,41	89,29
9	91,35	90,96	91,07
10	91,57	91,54	91,65
12	91,48	92,76	92,12
14	91,82	93,01	93,20
16	92,07	93,21	92,32
18	91,87	93,76	93,81
20	91,68	93,87	93,54
22	92,07	94,15	93,73
24	92,15	94,34	93,90
26	92,23	94,29	93,78
28	92,48	94,29	94,09
30	92,59	94,37	94,23
32	92,62	94,21	93,70
34	92,65	94,21	93,92
36	92,65	94,26	93,76
38	92,73	94,18	93,92
40	92,68	94,18	94,20
42	92,79	94,21	94,01
44	92,76	94,23	94,03
46	92,68	94,23	94,39
48	92,62	94,23	94,37
50	92,65	94,18	93,90
75	92,84	93,96	94,64
100	92,95	93,87	95,00
125	92,93	93,79	94,42
150	92,90	93,68	94,62
175	93,17	93,68	94,92
200	93,09	93,43	94,62

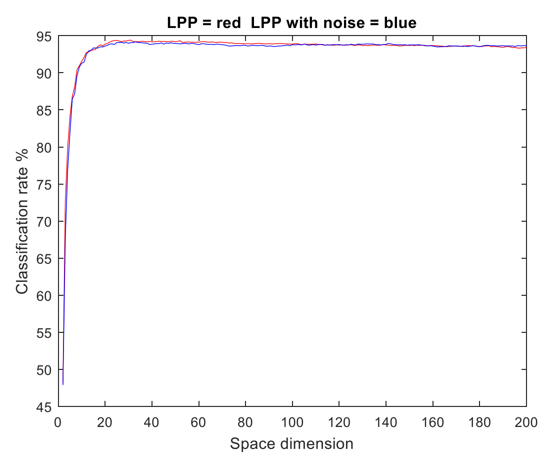

Fig. 2. Classification Rate % vs. Space Dimension with LPP (Sigma = 5, Neighborhood k = 9) Noisy and Noiseless ECG Normalised Patterns.

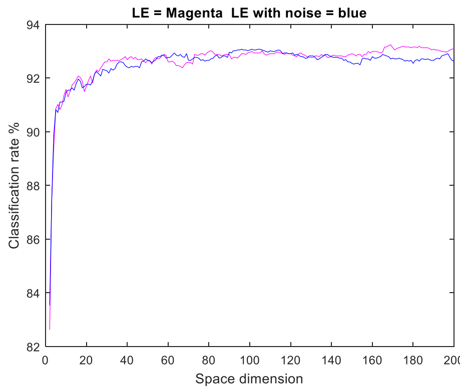

Fig. 3. Classification Rate % vs. Space Dimension with LE (Sigma = 5, Neighborhood k = 9) Noisy and Noiseless ECG Normalised Patterns.

For dimensions less than 10, it has been found that the classification rate for CS may differ, depending on the projection matrix. For this we tested several projection matrices; Table II presents these results and the average classification rates. For all three methods, the sensitivity of the algorithm to data normalization was analyzed as well.

Fig. 4 shows the classification rates obtained by compressed sensed ECG patterns with and without noise. The results are similar, so CS is not noise-sensitive as well.

Fig. 5 shows ECG patterns transformed into a 3-dimensional space for LE (87.46% classification rate), LPP (72.55% classification rate) and CS techniques (66.5% classification rate).

Fig. 6 shows the classification rate obtained by LPP for ECG segments with various levels of noise and without noise for non-normalized signals. Interestingly, for dimensions less than 40, the classification rates are practically unaffected by noise. However, for larger space dimensions, the results are worse but are improved by noise. Observe that, if we refer to the maximum values achieved in terms of classification, then LPP offers the best results for space dimension equal to 27 with a classification rate of 94%, even higher than that obtained for the initial non-normalized ECG signals (space dimension equal to 301 and classification rate 92.5%).

In Table III, several results for the LPP method applied to noiseless and noisy ECG waveforms are presented.

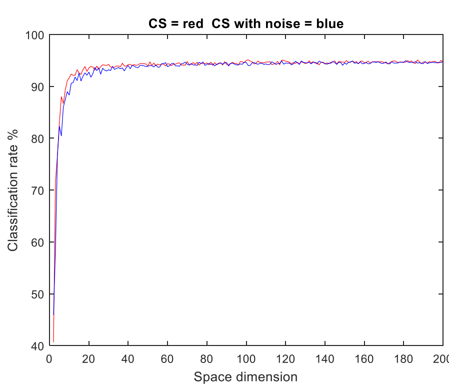

Fig. 4. Classification Rate % vs. Space Dimension with CS Noisy and Noiseless ECG Normalised Patterns.

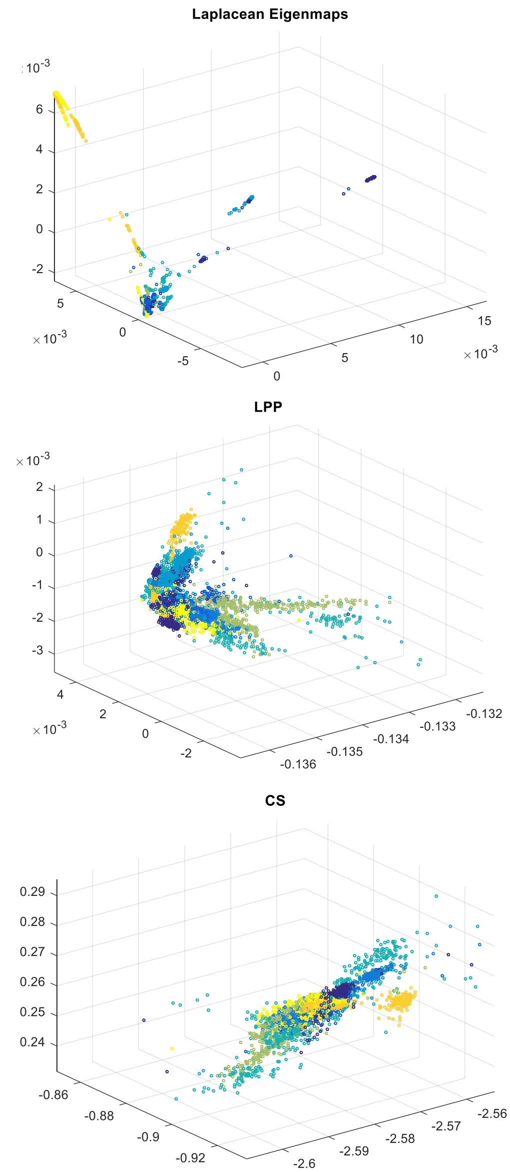

Fig. 5. ECG Patterns Reprezented into a 3-Dimensional Space with LE, LPP and CS Techniques.

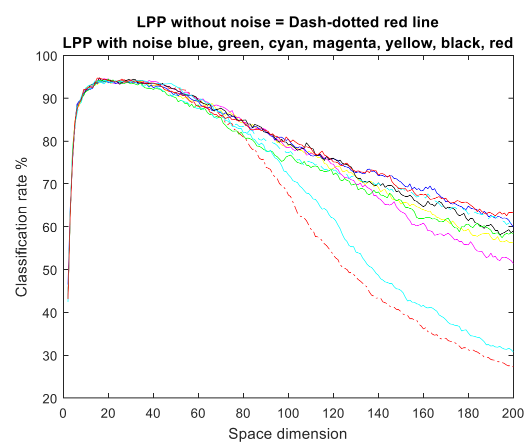

Fig. 6. Classification Rate % vs. Space Dimension with LPP (Sigma = 5, Neighborhood k = 9) for Original and Noisy Non-Normalised ECG Patterns.

TABLE. II. CLASSIFICATION RATE VS. SPACE DIMENSION FOR COMPRESSED SENSED WITH A FEW PROJECTION MATRICES

DIM	**mean**	**CS 1**	**CS 2**	**CS 3**	**CS 4**	**CS 5**	**CS 6**	**CS 7**	**CS 8**	**CS 9**	**CS 10**
2	**39,09**	44,64	33,19	36,56	37,50	40,33	40,75	38,47	36,53	42,25	40,69
3	**53,22**	53,31	54,94	50,42	54,81	49,72	48,44	61,31	48,03	64,39	46,86
4	**66,03**	66,19	63,50	66,81	68,83	62,50	68,83	63,78	74,22	62,06	63,56
5	**73,20**	74,53	80,19	74,31	71,97	70,08	69,75	74,56	68,00	72,50	76,11
7	**79,63**	83,47	81,17	74,28	78,89	77,94	83,58	80,44	77,92	76,33	82,22
9	**84,54**	84,89	84,31	84,17	84,25	85,31	84,92	85,03	84,78	82,42	85,31
11	**85,68**	85,56	82,44	85,47	84,94	86,28	87,25	87,42	87,28	85,94	84,19
13	**86,86**	85,11	88,39	89,03	85,72	85,44	87,75	85,56	86,97	88,11	86,50
15	**87,98**	87,94	87,06	87,56	88,36	88,00	87,31	88,28	87,83	88,94	88,53
17	**88,21**	89,50	88,56	89,33	87,78	87,67	87,67	88,22	87,69	88,42	87,22
20	**88,99**	89,75	89,00	90,03	88,72	88,75	89,92	88,86	88,14	88,75	87,94
25	**89,12**	89,72	87,94	89,00	87,94	89,14	89,94	89,81	90,42	88,72	88,58
30	**89,33**	89,69	88,83	89,31	88,92	89,00	88,75	89,86	90,50	89,00	89,42
35	**89,95**	90,14	89,86	89,36	89,64	90,56	90,19	90,56	89,81	89,19	90,19
40	**89,76**	90,14	89,75	89,44	89,11	90,83	89,25	89,17	90,17	89,78	89,94
45	**89,81**	89,83	89,50	89,94	90,17	89,14	90,19	89,97	89,44	89,72	90,17
50	**90,14**	90,50	89,31	89,94	90,22	89,86	90,64	89,89	90,25	90,83	90,00

TABLE. III. CLASSIFICATION RATE VS. SPACE DIMENSION FOR LPP WITH SEVERAL TYPES OF NOISE (FOR NON-NORMALISED SIGNALS)

Space Dim.	normal distribution Mean = 2 SD = 2	uniform distribution (-5, 5)	normal distribution Mean = 1 SD = 2	normal distribution Mean = 5 SD = 2	normal distribution Mean = 1 SD = 1	LPP without noise
	Red color	yellow color	black color	green color	blue color	dash-dotted red line
50	91.35	90.67	91.35	92.03	92.03	92.47
80	84.38	83.45	84.26	84.88	84.38	80.90
100	80.40	77.29	80.40	79.78	78.47	67.39
125	74.30	72.37	74.05	74.61	71.44	50.28
150	68.89	68.64	70.44	67.64	63.53	40.20
175	66.09	64.09	65.09	63.04	57.06	32.55
200	63.41	60.30	59.93	59.05	51.40	27.26

IV. CONCLUSIONS

The results presented in this paper concerned LE, LPP and CS dimension reduction techniques for ECG signals without and with noise.

Interestingly, for dimensions up to 10, the results obtained with LE are best even for very small dimensions like 2D or 3D. Even though the classification rates are smaller, it is possible to make an intuitive image on data separation. However an inconvenience of LE (unlike LPP) is that for each new data the computation should be taken from the beginning.

The results obtained with CS are close to those obtained with LE, CS offering the advantage of very low complexity in the compression stage. Another major advantage is that if a representation in which the signal is sparse is known, then from the reduced space it is possible to reconstruct (with some error) the initial data.

An advantage of LPP is that once the projection subspace has been found any new data entry will be projected on it with no other computation. However, although LPP has been successfully applied in numerous practical problems of pattern recognition the method may have some problems since the LPP results depend mainly on its underlying neighborhood graph whose construction suffers as the graph is constructed using the nearest neighbor criterion which tends to work weakly for high-dimensional original space and it is generally uneasy to assign appropriate values for the neighborhood size and heat kernel parameter implicated in the graph building.

In the case of LPP, we have found that the ECG classification results are influenced by the normalization of the signals for high space dimension-the influence of normalization is insignificant for space dimension less than 27. This can be justified by the fact that for the test signals the ECG signals are projected onto the new found subspace (the projection matrix represented by the corresponding eigenvector matrix).

Last, but not least, we found that of all three tested algorithms, LPP is the most robust to noise but sensitive to data normalization, while CS is sensitive to small dimensions of space at the projection of the matrix.

In the future, we will analyze the influence of data normalization on classification rates for dimensionality reduction methods.

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