Design of Robust Quasi Decentralized Type-2 Fuzzy Load Frequency Controller for Multi Area Power System

Jesraj Tataji Dundi[1], Rama Sudha Kasibhatla[3]
Department of Electrical Engineering
AUCOE (A), Andhra University
Visakhapatnam, India[1, 3]

Anand Gondesi [2]
Department of Electrical and Electronics Engineering
Dr.L.B College of Engineering
Visakhapatnam, India

A. Chandrasekhar [4]
Department of Engineering Mathematics
GITAM University
Visakhapatnam, India

***Abstract*—Interconnected power systems receive power through tie lines. Sudden perturbation in load causes uneven power distribution issues resulting in sudden changes in the voltage and frequency in the given system (tie-line power exchange error). The Load Frequency Controller (LFC) has an ability to stabilize the system for the above mentioned disturbances. In this paper, a novel load frequency controller based on Type-2 Fuzzy Quasi-Decentralized Functional Observer (T2FQFO) is proposed. In the proposed methodology the observer gains are obtained mathematically which guarantees, the stability of the system. The efficacy of the proposed technique has been tested on an IEEE standard testing systems. The results shows the proposed T2FQFO has higher performance when compared with Fuzzy Quasi-Decentralized Functional observer, Quasi- Decentralized Functional observer and classical state observer. And the results say that the peak over shoot and settling time have been improved by 25% (Approx.) by Type-2 Quasi Decentralized Functional Observer ((T2FQFO) than other observers.**

Keywords*—Load frequency control; type-2 fuzzy quasi-decentralized functional observer; fuzzy quasi-decentralized functional observers; state observer; type-2 fuzzy controller

I. Introduction

State Estimation and its analysis constitute a fundamental aspect of science and engineering. Information about the internal state of a given system can be retrieved simply by measuring the values of inputs and outputs to the system, under this process. The term system state stands for the internal condition of a system at any given instant of time. Dynamic State Estimation (DSE) [7] refers to analysis of internal characteristics that trigger changes in the given system. These are majorly applicable to most engineering and science disciplines (viz. Electrical, Electronics, Civil, Mechanical, Aerospace, Chemical, etc. [13].).

In Control System, an Observer refers to a sub-system that is employed for estimating the internal state/condition of another system (of which it is a part) solely by analyzing the input and output to the system. This idea was first introduced by D. Luenberger in 1966 [21]. Some of the commonly used observers are state observer, functional observer, sliding observer, bounding observer [4]. The terms Observer and State Observer are almost synonymous and are implemented using computers in industrial applications.

Functional observers on the other hand, work using probabilistic/statistical approaches. They are usually available in reduced forms. For LTI (linear time invariant) systems, [3] such observers can still be designed even if the system becomes sun-observable. The paper primarily discusses the application of the aforesaid technique for power systems where exposure to disturbances like sudden changes in the load, modification in system configuration, loss of transmission lines, generator failures, etc.[16][17] are common phenomena.

In this system, there is every possibility of random values creeping into the measured data set. Nevertheless, the state estimation technique being probabilistic in nature can be carried out with random constituents. Hence, probabilistic or set-membership approaches are preferred for data analysis [9]. The common schemes include Method of Moments Estimation, Minimum Variance Unbiased Estimator (MVUE), Maximum a Posterior (MAP), Maximum Likelihood Estimation (MLE), Bayes Least Squared Error (BLSE), Bayes Estimation, Nonlinear System Identification, Wiener Filter, Kalman Filter, Particle Filter [2][6].

Collection of various sub-systems that facilitate power supply or transfer is referred to as a power system. Multiple units/areas in a power system connected via transmission lines are known as interconnected power systems. State observers may be used to measure the magnitude of the transmitted power, the transmission lines being used are commonly known as tie-lines [5]. All units/areas must have identical line frequencies. Load frequency control (LFC) [18] method prevents deviations of frequency and tie-lines for all areas/units under steady state. The main objective of this paper is a comprehensive study of linear and non-linear dynamic state estimation techniques for fault detection and LFC using PMU’s [19]. A modern power system is constituted by various types of generators, transmission networks, measuring devices [8], etc. with properties similar to a general power system.

The paper describes the work (methodology) in Section II, the results obtained and the description in Section III and the conclusion in Section IV below.

A. Literature Review

M. Darouach et al. [1] introduced a simple method to design a full-order observer for linear systems with unknown inputs provided with the necessary and sufficient conditions for the existence of the observer. This method reduces the design procedure of full-order observers with unknown inputs to a standard one where the inputs are known. The existence conditions are given, and it was shown that these conditions are generally adopted for unknown inputs observer problem.

Chuang Liu, Hak-Keung Lam et al. [9] in their paper the authors investigated the stability of a Takagi-Sugano fuzzy model-based (FMB) functional observer control system. If the state-feedback control cannot measure the system state, the fuzzy function observer is designed to estimate the control input instead of directly estimating the system state. A fuzzy function observer can reduce the observer order, which determines a large number of observer gains.

Therefore, we propose a new form of fuzzy function observer that facilitates stability analysis so that the gain of the observer can be obtained numerically while guaranteeing stability. The proposed form also shows using the separation principle to design the fuzzy controller and the fuzzy function observer separately. To design fuzzy controllers with system stability in mind, we use higher derivatives of the Lyapunov function (HODLF) to reduce the conservativeness of the stability condition. HODLF generalizes the commonly used first derivative.

By exploiting the properties of the membership functions and the dynamics of the FMB control system, convex and relaxed stability conditions can be derived. A simulation example is provided to demonstrate the feasibility of the proposed relaxation of the stability constraint and the designed fuzzy function observer controller. Based on the proposed fuzzy function observer, operators can easily achieve stable observer gains. To confirm efficacy and mitigation, LIU et al. designed a fuzzy function observer controller for the nonlinear system 1639 via Holdf.

Further a more advanced techniques can be applied to meet the boundary conditions of the derivation of membership functions. And by extending the technique in discrete-time linear functional observer, the discrete-time fuzzy functional observer can also be investigated.

M. Darouach et al. [13], presented a simple and straightforward method to design full and reduced order observers for linear time-invariant descriptor systems and these are presented in this paper. The approach for the reduced order observer design is based on the generalized Sylvester equation. Sufficient conditions for the existence of the observers are given. An illustrative example is included. The reduced-order observer design method is based on the new resolution method of the constrained generalized Sylvester equation. It was shown that the existence conditions of the observer generalize those adopted in D. N. Shields et al. for the observer design of square descriptors systems problem. An extension to a less restrictive conditions is under study.

K. Rama Sudha et al. [16] have described a method based on the type 2 fuzzy system 'T2FS'. For LFC (Load Frequency Control) in power systems including Superconducting Magnetic Energy Storage Units (SMES) of two-area connected reheat heating systems. Therefore the author proposed a Type 2 (T2) fuzzy approach for load frequency control in a two-section connected reheat power plant, considering the Generation Rate Constraint (GRC), Boiler Dynamics (BD), and SMES. A distinct advantage of this controller is its lack of sensitivity to large load changes and system parameter variations, even in the presence of nonlinearities. The proposed method is tested in a dual-range power system to demonstrate its robust performance under different ranges of load changes. We compare the performance of a type 2 (T2) fuzzy controller with an optimal PID controller (Khamsum's optimal PID) and a fuzzy PI controller (type 1 fuzzy) controller in the presence of GRC, BD, and SMES. Simulation results confirm the high robustness of the proposed small-power-capacity SMES controller against various disturbances and system uncertainties compared with SMES in previous studies.

II. Power System Dse and Lfc using Functional Observers

Application of Quasi decentralized functional observer (QDFO) to two area inter-connected linear power system with a single tie-line model has been depicted [11]. Development of Functional Observer (FO) [3] model for highly interconnected power systems have been under taken and is presented. The LFC signal has been generated here using a control signal. The signal needed is acquired directly from the functional observer (FO) instead of forming a control signal out of a linear combination of individual state signals. Tie-line power measurements, voltage magnitudes along with phase angles and current measurements combined with current voltage measurements of the PMU are employed for the Quasi decentralized functional observer. Power system state is primarily analyzed using an estimation algorithm, ‘observability’ analysis, bad data recognition etc. [7]. The basic idea and its analysis were formulated in this paper, under five corollaries.

A. Corollary -I

It might not be possible to measure all state variables under practical circumstances either due to complications related to data acquisition or cost issues. As a result, for all states of the state vector, a state estimation technique is required to obtain state feedback [19].

Let $\hat{x}(t)$ = estimate of the state vector $x(t)$

State Observer/Estimator

$$\dot{x} = \alpha x + \beta u \tag{1}$$

$$y = \gamma x \tag{2}$$

Here x serves as the state vector, y is the output (which includes an estimation of the state vector x).

The control signal is represented by ‘u’.

Open-loop Observer (L= [0], αobs=α)

$$\dot{\hat{x}}(t) = \alpha\hat{x}(t) + \beta u \tag{3}$$

Failure of the observer takes place due to disturbances and modeling error.

Open-Loop Estimation Error

$$\tilde{x}(t) = x(t) - \hat{x}(t) \quad (4)$$

$$\dot{\tilde{x}}(t) = \dot{x}(t) + \dot{\hat{x}}(t) = \alpha\tilde{x}(t) \quad (5)$$

Hence,

$$\tilde{x}(t) = e^{\alpha t}\tilde{x}(0) \quad (6)$$

Error Dynamics:

There is no rectification for modeling imperfections, and state matrix α is unstable and has unbounded error.

$$\dot{\hat{x}} = \alpha\hat{x} + \beta u + \mathcal{L}(y - \gamma\hat{x}) = \alpha\hat{x} + \beta u + \mathcal{L}y \quad (7)$$

$$\alpha_{obs} = \alpha - \mathcal{L}\gamma \quad (8)$$

Where $\mathscr{L}$ is the observer gain

Eigen values are assigned to the matrix $\mathcal{L}$. Appropriate estimation requires the equation to be reduced in terms of an open loop observer as shown in the Fig. 1.

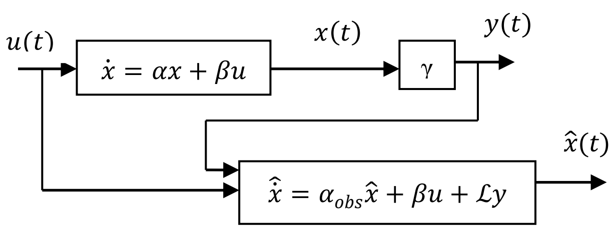

Fig. 1. Block Diagram of Observer.

Theorem1:

The system (γ, α) is observable if and only if the dual system (α^T, γ^T) is controllable [19].

Eigen values could be arbitrarily assigned (stable) by using state feedback provided (α^T, γ^T) is controllable.

$$\dot{x} = \alpha^T x + \gamma^T u \quad (9)$$

$$u = -\mathcal{L}^T x \quad (10)$$

$$\dot{x} = (\alpha^T - \gamma^T\mathcal{L}^T)x \quad (11)$$

$$(\alpha^T - \gamma^T\mathcal{L}^T)^T = \alpha - \mathcal{L}\gamma \quad (12)$$

Similar Eigen values are obtained.

B. *Corollary -II*

In application to the load frequency control, Functional Observability is better than State Observability [20].

Functional observer can be stated as follows.

$$\dot{x}(t) = \alpha x(t) + \beta u(t) \quad (13)$$

$$y(t) = \gamma x(t) \quad (14)$$

$$z(t) = \mathcal{L}x(t) \quad (15)$$

Here $x(t) \in R^n, u(t) \in R^m$ are the state input vectors, $y(t) \in R^p$ is the state output vector and $z(t) \in R^r$ is the vector to be estimated. $\alpha \in R^{nxn}$, $\beta \in R^{nxm}$, $\gamma \in R^{pxn}$ and $\mathcal{L} \in R^{rxn}$ are matrices of known constants. A functional observer should be a dynamic system that should be able to track z(t) asymptotically. It is theorized to have the following structure:

$$\dot{w}(t) = \eta w(t) + \lambda y(t) + \mu u(t) \quad (16)$$

$$\hat{z}(t) = gw(t) + €y(t) \quad (17)$$

α, β and γ are the system matrices and η, λ, μ, D and € are the observer matrices and which are defined as:

$\alpha \in R^{nxn}$ N$\in R^{qxq}$

$\beta \in R^{nxm}$ $\lambda \in R^{qxp}$

$\gamma \in R^{pxn}$ $\mu \in R^{qxm}$

$\mathscr{L} \in R^{rxn}$ D$\in R^{rxp}$ €$\in R^{rxp}$

Theorem 2:

If and only if the following conditions are met, the completely observable qth order functional observer of (16) and (17) will estimate Lx(t).

- A stability matrix,η

$$\lambda\gamma = P\alpha - \eta P \quad (18)$$

$$\mu = P\beta \quad (19)$$

$$\mathcal{L} = DP + €\gamma \quad (20)$$

Observer error in state estimation can be defined as

$$e(t) \triangleq w(t) - Px(t) \quad (21)$$

Derivation and upon substitution of the system and observer equations, we get

$$\dot{e}(t) = w(t) + P\dot{x}(t) = \eta w(t) + \lambda\gamma\dot{x}(t) + \mu u(t) - P\alpha x(t) - P\beta u(t) \quad (22)$$

Applying conditions (13) and (14) yields

$$\dot{e}(t) = \eta e(t) \quad (23)$$

The differential solution above is in the form of an exponential function

$$e(t) = e^{\eta t} \quad (24)$$

in which the observer dynamics are controlled by the variable N.

Applying these conditions, we get:

$$\lim_{t\to\infty} e(t) = w(t) - Px(t) \quad (25)$$

Further simplification, leads to the following we get

$$e_z(t) = \hat{z}(t) - \mathcal{L}x(t) = D(w(t) - Px(t)) \quad (26)$$

The above equation should reach zero asymptotically as expected α and η do not share common Eigen values but confirms that P will have a unique solution. Now X(t) and e(t) are easily derived by using above defined conditions.

$$\dot{x}(t) = \alpha x(t) + \beta u(t) = \alpha x(t) - \beta(Dw(t) + €y(t))$$
$$= (\alpha + \beta\mathcal{L})x(t) + (\beta D)e(t) \quad (27)$$

$$\dot{e}(t) = \eta e(t) \quad (28)$$

This results in a composite system similar to the full-state observer as

$$\begin{bmatrix} \dot{x}(t) \\ \dot{e}(t) \end{bmatrix} = \begin{bmatrix} \alpha + \beta\mathcal{L} & \beta D \\ 0 & \eta \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} \quad (29)$$

Aside from the fact that the control law, $\mathcal{L}x(t)$ is used instead of -Kx(t).Though there is variation in notation, they are similar to each other. Judging the constraints involved, an r^{th} order observer is required to achieve functional state of the system, where the order of the system r is as small as possible. The observer matrices should therefore guarantee the ease of Eigen value assignment and simplicity of the control algorithm so that it can be readily applied.

For order estimation, the matrix ranks are taken into consideration.

$$\text{rank}\begin{bmatrix} L\alpha \\ \gamma\alpha \\ \gamma \\ \mathcal{L} \end{bmatrix} = \text{rank}\begin{bmatrix} \gamma\alpha \\ \gamma \\ \mathcal{L} \end{bmatrix} \quad (30)$$

$$\text{rank}\begin{bmatrix} s\mathcal{L} - \mathcal{L}\alpha \\ \gamma \\ \mathcal{L} \end{bmatrix} = \text{rank}\begin{bmatrix} \gamma\alpha \\ \gamma \\ \mathcal{L} \end{bmatrix} \quad s \in \gamma, R(s) \geq 0 \quad (31)$$

The condition is satisfied when the ranks on the LHS and RHS are equal. The author in [15] shows that this condition is equal to the delectability of the pair (F, g), where

$$F = \mathcal{L}\alpha\mathcal{L}^{+} - \mathcal{L}\alpha(I - \mathcal{L}^{+}\mathcal{L}) \begin{bmatrix} \gamma\alpha(I - \mathcal{L}^{+}\mathcal{L}) \\ \gamma(I - \mathcal{L}^{+}\mathcal{L}) \end{bmatrix}^{+} \begin{bmatrix} \gamma\alpha\mathcal{L}^{+} \\ \gamma\mathcal{L}^{+} \end{bmatrix} \quad (32)$$

$$g = \left(I - \begin{bmatrix} \gamma\alpha(I - \mathcal{L}^{+}\mathcal{L}) \\ \gamma(I - \mathcal{L}^{+}\mathcal{L}) \end{bmatrix} \begin{bmatrix} \gamma\alpha(I - \mathcal{L}^{+}\mathcal{L}) \\ \gamma(I - \mathcal{L}^{+}\mathcal{L}) \end{bmatrix}^{+} \right) \begin{bmatrix} \gamma\alpha\mathcal{L}^{+} \\ \gamma\mathcal{L}^{+} \end{bmatrix} \quad (33)$$

Where, $\mathcal{L}$+ denotes the Moore-Penrose generalized inverse of matrix $\mathcal{L}$. Moreover, if matrices λ,μ and E satisfy Theorem 1, a Hurwitz matrix η is given by

$$\eta = F - Zg \quad (34)$$

For F-Zg to be stable, the Matrix Z is obtained by any pole placement method. Matrices E and K are obtained according to the equation,

$$[E \quad K] = \mathcal{L}\bar{\alpha}\Sigma^{+} + Z(I - \Sigma\Sigma^{+}) \quad (35)$$

Where, $\bar{\alpha} = \alpha(I - \mathcal{L}^{+}\mathcal{L})$, $\bar{\gamma} = \gamma(I - \mathcal{L}^{+}\mathcal{L})$ and $\Sigma = \begin{bmatrix} C\bar{A} \\ \bar{C} \end{bmatrix}$

Matrix λ, μ are obtained according to

$$\lambda = K + \eta E \quad (36)$$

$$\mu = (\mathcal{L} - E\gamma)\beta \quad (37)$$

All of the required observer parameters can be easily computed using this algorithm, which results in a functional observer of form

$$\dot{w}(t) = \eta w(t) + \lambda y(t) + \mu u(t) \quad (38)$$

$$\hat{z}(t) = w(t) + Ey(t) \quad (39)$$

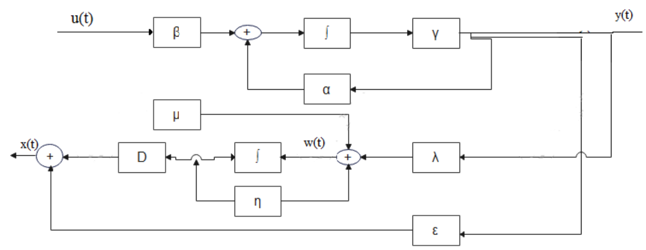

Fig. 2. Schematic of a Functional Observer.

C. Corollary –III

Functional Observer based Conventional Controller:

Due to increasing complexity in power distribution network, a simplified assumption that considering all the generators in a given area into one single generation unit, transmission lines and various bus bars are lumped into one single entity might not be appropriate enough for any complex power system network. This paper presents an analysis of a Quasi-decentralized Functional Observer (QDFO) project to control the tie-line power and frequency of multi-area interconnected power system with real time considerations [10]. Also, linear system (two-area) connected with a single tie-line model was considered for the generation of control signals and Quasi-Decentralized Functional Observer(QDFO) is applied to it for the control. Further it is formulated with FO [3] approach for LFC (Load Frequency Control) of highly interconnected power networks. In the process of generation of the LFC signal, it is required to estimate the control signal. It is more rational to estimate the desired signal directly using the functional observer (FO) than estimating all the individual states and then linearly combining those individual state estimates to construct the control signal [4]. And the control signal is generated by a Quasi-Decentralized Functional Observer (QDFO) in this case. PMU measurements of voltage and current magnitudes and phase angles, as well as tie-line power measurements, are used in the Functional Observer (FO) estimate process [2]. The suggested FO-based controllers have less complicated architecture and comparable performance to Full Order observers. Further, the Functional Observability (FO) criterion is less rigorous than the state observability requirement, and the design and analysis of the observer of overall network topology [9] is take into account.

Design Algorithm

1) L is Partitioned according to $\mathcal{L} = \begin{bmatrix} \mathcal{L}_1 \\ \mathcal{L}_2 \end{bmatrix} = \begin{bmatrix} g\gamma \\ \mathcal{L}_2 \end{bmatrix}$

2) Checking if condition (26) is met, if yes continue, otherwise a sliding mode functional observer doesn't exist.

3) Calculating *F* using (32) and *G* using (33).

4) Using (34) and any pole placement method obtain *Z* to make η Hurwitz.

5) Calculate *E* and *K* using (35)

6) Use (36) to calculate λ.

7) Calculate μ according to (37).

Design of FO based Controller:

Let $\hat{e}$ be the estimate of e.

$$\dot{\hat{e}} = \alpha\hat{e} - \beta\Gamma_c^T\hat{e} + \Gamma_o(e_1 - \hat{e}_1) \quad (40)$$

Where, $\Gamma_o \equiv [\gamma_1^o, \gamma_2^o, \ldots, \gamma_n^o]^T \epsilon R^n$ is the observer gain vector.

Observer error is given as,

$$\tilde{e} = \alpha_o\tilde{e} + \beta(g(x)(u^* - u_{PID} - u_d - u_s) - d) \quad (41)$$

As (γ, α) is observable, the observer gain vector,$\Gamma_o$can be stringently Hurwitz with a symmetric positive definite matrix,P and a positive definite matrix, Q_o.

If $\mathcal{L}(.)$ is taken to be the Laplace transform, the transform function can be written as $\mathcal{L}(\tilde{e}_1)$ by choosing $\mathcal{M}(s)$ such that $\mathcal{M}(s) = s^m + b_1s^{m-1} + \cdots + b_m$, $m < n$ and $\mathcal{M}^{-1}(s)$ is a proper stable transfer function and $\eta(s)\mathcal{M}(s)$ is a proper SPR transfer function. Hence $\mathcal{L}(\tilde{e}_1)$ can be written as:

$$\mathcal{L}(\tilde{e}_1) = \eta(s)\mathcal{M}(s)\mathcal{M}^{-1}(s)\mathcal{L}(g(x)(u^* - u_{PID}) - d) - \eta(s)\mathcal{M}(s)\mathcal{M}^{-1}(s)\mathcal{L}(g(x)u_d) - \eta(s)\mathcal{M}(s)\mathcal{M}^{-1}(s)\mathcal{L}(g(x)u_s) + \eta(s)\mathcal{M}(s)\mathcal{L}(u^* - u_{PID}) - d) - \mathcal{L}(u^* - u_{PID}) \quad (42)$$

Denote the function φ such that

$$\mathcal{L}(\varphi) = \mathcal{M}(s)^{-1}\mathcal{L}(g(x)(u^* - u_{PID}) - d) - \mathcal{L}(u^* - u_{PID}),$$

Hence the dynamce equation can be written as

$$\tilde{e}_1 = \gamma_m^T\tilde{e} \quad (43)$$

Where, $B_m = [0,0, \ldots, 0,0, \ldots, b_1, b_2, \ldots b_m]^T \epsilon \Re^n$,

$C_m = [1,0, \ldots 0]^T \epsilon \Re^n$

For further analysis, there is necessity to assume the following assumptions

Assumption 1: The uncertain non-linear function f(x) for the states is bounded by an upper bound function$f^u(x)$, i.e. $f(x) \leq f^u(x)$. The uncertain non-linear function g(x) related with the input is bounded by $g_l \leq \| g(x) \| \leq g^u$ where both upper and lower boundaries g^u and g_l are positive constants.

Assumption 2: the function φ is bounded by $\| \varphi \| \leq \varepsilon$ where ε is a positive constant.

Defining and differentiating V with respect to t, we get

$$\dot{V} = \frac{1}{2}\Big((\tilde{e}^T\alpha_0^TP\tilde{e} + \tilde{e}^TP\alpha_0\tilde{e}) + (u^* - u_{PID} + \varphi - \tilde{u}_d - \tilde{u}_s)^T\beta_m^TP\tilde{e} + \tilde{e}^TP\beta_m(u^* - u_{PID} + \varphi - \tilde{u}_d - \tilde{u}_s)\Big) \quad (44)$$

But it is given that,

$$\alpha_0^TP + P\alpha_0 = -Q \quad (45)$$

And $P\beta_m = \gamma_m$ (46)

Where, $Q = Q^T > 0$. Substituting (45), (46) and (43) in (44) we get,

$$V \leq -\frac{1}{2}|\tilde{e}^TQ\tilde{e}| + |e_1||u^* - u_{PID}| + \tilde{e}_1(\varphi - \bar{u}_d) - \tilde{e}_1\bar{u}_s \quad (47)$$

Based on the above assumptions, u_d can be designed such that $\tilde{e}_1(\varphi - \bar{u}_d) \leq 0$.

$$u_d = \begin{cases} \varepsilon + k, & \text{if } \tilde{e}\_1 \geq 0 \text{ and } \varphi > 0 \\ 0 & \text{if } \tilde{e}_1 \geq 0 \text{ and } \varphi < 0 \\ 0, & \text{if } \tilde{e}_1 \geq 0 \text{ and } \varphi > 0 \\ -(\varepsilon + k), & \text{if } \tilde{e}_1 \geq 0 \text{ and } \varphi < 0 \end{cases} \quad (48)$$

Where, k is a positive constant. Substituting $u_d$in (47),

$$\dot{V} \leq -\frac{1}{2}|\tilde{e}^TQ\tilde{e}| + |\tilde{e}_1||u^* - u_{PID}| - \tilde{e}_1\tilde{u}_s$$
$$\leq -\frac{1}{2}\lambda_{min}(Q)|\tilde{e}_1|^2 + |\tilde{e}_1||u^* - u_{PID}| - \tilde{e}_1\tilde{u}_s \quad (49)$$

$\dot{V} \leq$
$-1/2\ \lambda\_min\ (Q)\ |e\tilde{}\_1\ |\hat{}2 + |e\tilde{}\_1\ |(1/|g\_1\ |\ \ (|f\hat{}u\ (x\hat{})| + |y\_m\hat{}((n))\ | + |\Gamma\_e\hat{}Te\hat{}\ |\ ) + |u\_PID\ |\ ) - e\tilde{}\_1\ u\_s\ )$ (50)

$$V_d(K_{PID}) \leq -\frac{1}{2}\lambda_{min}(Q)|\tilde{e}_1|^2 + |\tilde{e}_1|\left(\frac{1}{|g_1|}\left(|f^u(\hat{x})| + \left|y_m^{(n)}\right| + |\Gamma_e^T\hat{e}|\right) + |u_{PID}|\right) \quad (51)$$

If K_{PID} estimated by MGA results in $V_d(K_{PID}) < 0$, $\dot{V} < 0$ is satisfied: given that u_s is not applied to the input in (10). Conversely, if $V_d(K_{PID}) > 0$,u_s should be applied leading to the condition $\dot{V} < 0$. For stabilizing the effect caused by u_s being either excluded or included in PID controller, define a gate function.

$$\theta(V_d) = \begin{cases} 0, & \text{if } V_d < 0; \\ \frac{V_d}{V_m}, & \text{if } 0 \leq V_d < V_m \\ 1, & \text{if } V_d \geq V_m \end{cases} \quad (52)$$

Where, V_m is a positive constant. Now the supervisory control with the gate function becomes

$$u_s \leq \theta(V_d)\mathrm{sgn}(\tilde{e}_1)\left(\frac{1}{|g_1|}\left(|f^u(\hat{x})| + \left|y_m^{(n)}\right| + |\Gamma_e^T\hat{e}|\right) + |u_{PID}|\right) \quad (53)$$

D. Corollary IV

In recent years, elements like heuristics, reasoning gained importance due to their ease in more flexible control theories. Fuzzy logic (FL), optimization algorithms like Genetic Algorithm (GA) and Artificial Neural Network (ANN) may be applied for further refinement of this model to facilitate the development of advanced/intelligent controllers. Fuzzy logic (FL) is advantageous in terms of its logical designing and decision making but its drawback lies in the fact that the solution takes a complex form if more number of variables are involved. To estimate the control input specifically when the output y is measurable instead of system state x [15], the author proposed Fuzzy Functional Observer [14]. The T–S fuzzy model (1) is assumed to be as [9]:

$$\dot{x} = \sum_{i=1}^{p} w_i(y)(\alpha_i x + \beta_i\breve{u}) \quad (54)$$

$$y = \gamma x \quad (55)$$

Where, $y \in R^l$is the output of the system, $\gamma \in R^{l\times n}$is the output matrix. Now the Fuzzy controller becomes:

$$u = \sum_{j=1}^{p} w_i(y)u_j = \sum_{j=1}^{p} w_i(y)g_j x \quad (56)$$

Where, $u_j = gG_j x \in R^m$ is the control input in j^{th} rule.

Assuming rank (γ) = l, rank (g_j) = m, i.e.,$\gamma$and $g_j$are of full row rank, the following fuzzy functional observer is proposed to estimate the control input u:

$$\dot{z}_j = \sum_{i=1}^{p} w_i(y)\left(\eta_{ij} z_j + \lambda_{ij} y + \mu_{ij} \breve{u}\right) \forall j \quad (57)$$

$$\breve{u}_j = z_j + E_j y \forall j \quad (58)$$

$$\breve{u} = \sum_{j=1}^{p} w_j(y)\breve{u}_j \quad (59)$$

Where, $z_j \in R^m, \breve{u}_j \in R^m$, $\breve{u} \in R^m$ are the observer state, estimated control input in j^{th} rule, estimated control input respectively; $\eta_{ij} \in R^{m\times m}$, $\lambda_{ij} \in R^{m\times l}$, $\mu_{ij} \in R^{m\times m}$, $E_j \in R^{m\times l}$are the observer gains to be designed.

Estimation error

$$e_j = Q_j x - z_j \text{ , where } Q_j = g_j - E_j \gamma \quad (60)$$

$$\dot{x} = \sum_{i=1}^{p} \sum_{l=1}^{p} h_{il}(\alpha_i x + \beta_i G_l x - \beta_i \sum_{k=1}^{p} w_k e_k) \quad (61)$$

$$\dot{e}_j = \sum_{i=1}^{p} \sum_{l=1}^{p} h_{il} \left((\phi_{ij} + \alpha_{ij} g_l)x + \eta_{ij} e_j - \alpha_{ij} \sum_{k=1}^{p} w_k e_k\right) \forall j \quad (62)$$

Where $h_{il} = w_i w_l$, $\phi_{ij} = Q_j \alpha_i - \eta_{ij} Q_j - J_{ij}\gamma$, $\alpha_{ij} = Q_j \beta_i - \mu_{ij}$

To make the system asymptotically stable, g_j , η_{ij} , λ_{ij}, $\mu_{ij}$and E_j is to be determine.

Now on imposing the constraints, we get

$$\dot{x}_a = \sum_{i=1}^{p} \sum_{l=1}^{p} h_{il} \Gamma_{il} x_a \quad (63)$$

$$\text{Where,} \Gamma_{il} = \begin{bmatrix} \alpha_i + \beta_i g_l & -\beta_i w_1 & -\beta_i w_2 \\ 0 & \eta_{i1} & 0 \\ 0 & 0 & \eta_{i2} \end{bmatrix}$$

Theorem 3:

The error systems are to be asymptotically stable if there are matrices, $X^T \in R^{m\times m}$, $Y_{ij} \in R^{m\times 2l}$, i,j=1, 2, . . . , p such that it satisfies the following:

$$X > 0$$
$$XF_{ij} - Y_{ij}M_{ij} + F_{ij}^T X - M_{ij}^T Y_{ij}^T < 0 \quad \forall i, j$$

$$\widetilde{E}_{i_1 j} = \widetilde{E}_{i_2 j} \forall i_1 < i_2, j$$

Where,

$$F_{ij} = g_j \alpha_i g_j^+ - g_j \overline{\alpha}_{ij} \Sigma_{ij}^+ \begin{bmatrix} \gamma \alpha_i g_j^+ \\ \gamma g_j^+ \end{bmatrix} \quad (64)$$

$$\mathcal{M}_{ij} = \left(I - \Sigma_{ij}\Sigma_{ij}^+\right) \begin{bmatrix} \gamma \alpha_i g_j^+ \\ \gamma g_j^+ \end{bmatrix} \quad (65)$$

$$\overline{\alpha}_{ij} = \alpha_i\left(I - g_j^+ g_j\right) \quad (66)$$

$$\Sigma_{ij} = \begin{bmatrix} \gamma \overline{\alpha}_{ij} \\ \overline{\gamma}_j \end{bmatrix} \quad (67)$$

$$\overline{\gamma}_j = \gamma\left(I - g_j^+ g_j\right) \quad (68)$$

Now the controller gain G_j can determineby

$$\left[\widetilde{E}_{ij} \quad \widetilde{K}_{ij}\right] = X g_j \overline{\alpha}_{ij} \Sigma_{ij}^+ + Y_{ij}\left(I - \Sigma_{ij}\Sigma_{ij}^+\right); \; Z_{ij} = X^{-1} Y_{ij} \quad (69)$$

Using the equation of constraint, we can have

$$N_{ij} = g_j \alpha_i g_j^+ - E_j \gamma \alpha_i g_j^+ - \left(\lambda_{ij} - \eta_{ij} E_j\right)\gamma g_j^+ \forall i, j$$

$$K_{ij} = \lambda_{ij} - \eta_{ij} E_j \quad (70)$$

The error system now becomes

$$\dot{e}_j = \sum_{i=1}^{p} w_i\left(F_{ij} - Z_{ij}\mathcal{M}_{ij}\right)e_j \forall j \quad (71)$$

Applying the Lyapunov function, time derivative of $\dot{V}(e_j)$is obtain as:

$$\dot{V}(e_j) = \sum_{i=1}^{p} w_i e_j^T \left(XF_{ij} - Y_{ij}\mathcal{M}_{ij} + F_{ij}^T X - \mathcal{M}_{ij}^T Y_{ij}^T\right)e_j \quad (72)$$

Where $Y_{ij} = XZ_{ij}$.

In order to determine the observer gains satisfying the constraints, it is necessary to calculate the following terms from the above equations:

1) η_{ij}
2) E_j and the intermediate variable K_{ij}
3) λ_{ij}
4) μ_{ij}.

Hence observer gains can be obtained.

E. Corollary V

Fuzzy Functional Observer Vs Type-2 Fuzzy Functional Observer:

Type-2 fuzzy systems can model complex uncertainties better than type-1 and it is important to develop type-2 fuzzy systems [12] for enhanced performance. Literature shows that type-2 fuzzy systems gained much importance in recent times. In this paper, T2FFO based on Lyapunov theorem have been modeled to investigate closed-loop stability. Numerous techniques have been presented in this paper for tuning free parameters in the design of optimal type-2 fuzzy systems. Few techniques like instance recursive orthogonal least-squares algorithm, multi-objective genetic optimization [3], steepest descent method, etc. can be employed. Taking the computational cost into account, as the type-reduction part is more, simplified type-2 fuzzy systems have been proposed. Also the direct defuzzification for type-2 fuzzy systems has been proposed and compared with type-reduction. The design of adaptive fuzzy controllers have some challenges like approximation error effects, computational cost, external disturbances and state estimation errors unswervingly on the stability of closed-loop system. Hence a novel robust observer for a class of uncertain nonlinear systems using a new adaptive compensator based indirect adaptive type-2 fuzzy controller [12] is presented to eliminate the disturbances. As it is being employed 3-dimensional type-2 membership functions, the “course of dimensionality" problem can be solved and this controller can be applied to the higher order systems.

Fuzzy systems with normal, orthogonal, consistent and complete triangular membership function in the antecedent. The systems are dynamic and their consequents are in the form of state equation and output equation. The system rules are in the form

$$R^{i_1\ldots i_m} \text{ IF } u(k) \text{is } \tilde{U}^{i_1\ldots i_m} \text{THEN}$$
$$\begin{cases} x^{i_1\ldots i_m}(k+1) = F^{i_1\ldots i_m}.x(k) + G^{i_1\ldots i_m}.u(k) \\ \qquad y(k) = C^{i_1\ldots i_m}.x(k) + (x(k),u(k)) \end{cases} \tag{73}$$

With $\tilde{U}^{i_1\ldots i_m}$ orthogonal triangular type-2 membership functions such that, $\tilde{U}^{i_1\ldots i_m} = [\tilde{U}_1^{i_1} \ldots \ldots \tilde{U}_m^{i_m}]^T$. The state vector is $x(k) = [x_1(k) \ldots \ldots . x_n(k)]^T$, the fuzzy input vector is $u(k) = [u_1(k) \ldots \ldots . u_m(k)]^T$,and the output vector is $y(k) = [y_1(k) \ldots \ldots . y_p(k)]^T$, $m \le n$, $i_j = 1,\ldots,N_j$, $j = 1,\ldots,n$. The constant matrices in the consequents are$F^{i_1\ldots i_m} \in \mathbb{R}^{n\times n}, G^{i_1\ldots i_m} \in \mathbb{R}^{n\times m}, C^{i_1\ldots i_m} \in \mathbb{R}^{p\times n}$ and the uncertainty in output is given by the state vector function and control input

$$D(x(k),u(k) = [D_1(x(k),u(k)),\ldots\ldots,D_p(x(k),u(k))]^T \tag{74}$$

Without loss of generality, $D_j, j = 1,\ldots.,p$, are fuzzy sets with membership functions is assume.

$$\mu_{D_j}(d_j) = $$
$$\begin{cases} \text{nonzero}, \; \underline{S}_j\left(x_j(k)\right) \le d_j \le \overline{S_j}\left(x_j(k)\right), \; j = 1,\ldots\ldots,n \\ \quad 0 \;, \qquad\qquad\qquad\qquad \text{elsewhere} \end{cases} \tag{75}$$

Type-2 Fuzzy PID controller with 3 input Type-2 Fuzzy Logic Controller (T2FLC) structure with coupled rules is shown in Fig. 3.

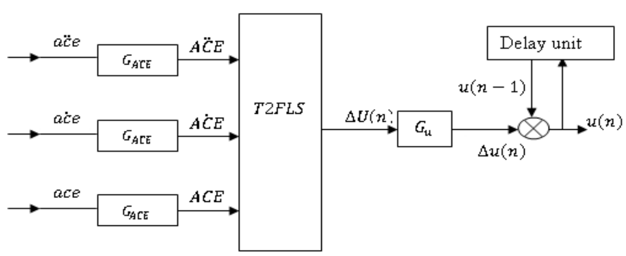

Fig. 3. Structure of the Interval Type-2 Fuzzy PID Controller.

Here ACE, $(ACE)^{\cdot}$ and $(ACE)^{\cdot\cdot}$ are the three input variables which are fuzzified by two interval type-2 fuzzy as positive and negative as shown below in Fig. 2 and represented by and Membership functions for ACE are given as:

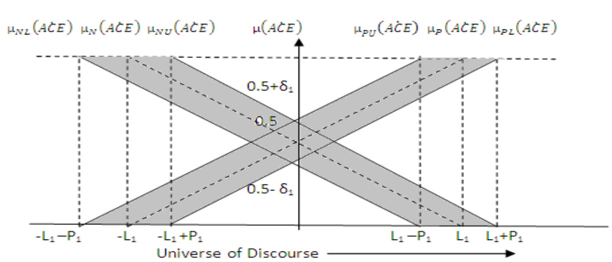

Fig. 4. Membership Functions of the Interval Type-2 Fuzzy Sets for the Scaled Input Variables.

TABLE I. CONTROL RULES FOR T2 FUZZY CONTROLLER

	ΔACE			
$\Delta A\dot{C}E$	N	P	N	N
	Z	N	P	P
	P	N	N	N

In this paper it is assumed that membership functions of the antecedents of all rules are consistent, triangular, complete and orthogonal (Fig. 4).

The block diagram of the plant and observer is as shown in Fig. 5 where $u(k)$, $y(k)$ and $\hat{x}(k)$ denotes the plant input, output and state estimate respectively. The system input is a fuzzy variable and the system output is crisp and fuzzy for systems.

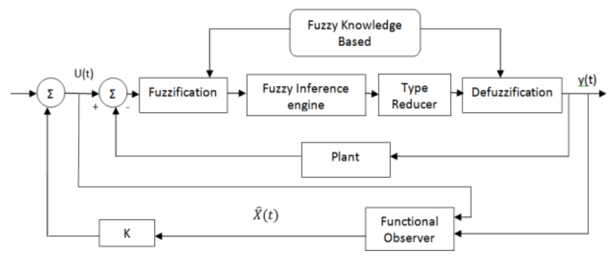

Fig. 5. Block Diagram of Plant and Observer.

III. RESULTS AND DISCUSSIONS

Simulation of the proposed technique has been carried out using Matlab / Simulink with an increase in demand of the first area ΔPD_1 and second area ΔPD_2. Testing with wide perturbations in input have been also carried out over both the areas i.e., demand of the first area ΔPD_1 and the second area ΔPD_2. It is apparent from the Fig. 6 – Fig. 12 that the system response is faster in terms of control and frequency deviations using proposed methodology are also nullified as a result. Thus the theorized model provides better control and frequency damping when performance comparison is made with fuzzy functional observer, functional observer, functional observer without controller and Luenberger/state observer, under all operating conditions.

Table I lists down the robustness of performance under various operating conditions numerically. Here the settling time, overshoot and undershoot have been calculated for different operating points. The Simulation results are shown for 10% band of step load change for operating point of Appendix A. According to Table I, the proposed T2FQFO based controller is better compared to the FQFO based controller, Functional Observer based controller, Functional Observer and the Luenberger observer based controller.

A. *Result Analysis:*

The Peak over Shoot, Undershoot and settling time at different operating points with different observers have been tabulated in the Table II shown below. From the table we can say that, Peak over shoot had been improved by 25.37% (approximately) and the settling time have improved by 8% using Type-2 Quasi Decentralized Functional Observer over Fuzzy Decentralized Functional Observer. Peak over shoot and the settling time had been improved by 46.77% (approximately) and 28.52% respectively by using Type-2 Quasi Decentralized Functional Observer over Fuzzy Decentralized Functional Observer. Peak over shoot and the settling time had been improved by 58.91% (approximately) and 47.7% respectively by using Type-2 Quasi Decentralized Functional Observer over Quasi Decentralized Functional Observer. Peak over shoot and the settling time had been improved by 53.28% (approximately) and 62.98% respectively by using Type-2 Quasi Decentralized Functional Observer over Full order Luenberger Observer. These parameters are taken at different operating points and these parameters are improving at different operating points. The parameters are more improved at operating point "5".

TABLE II. ΔF_1 (T) RESPONSE PERFORMANCE IN VARIOUS CONTROL STRATEGIES

Operating Point	Controller	Over Shoot (P.U)	Under Shoot (P.U)	Settling time(sec)
1	Type-2 Quasi Decentralized Functional Observer	0.05013	-0.06942	3.401
	Fuzzy Decentralized Functional Observer	0.06718	-0.0868	3.694
	Quasi Decentralized Functional Observer	0.09419	-0.0990	4.758
	Full order Luenberger Observer	0.1220	-0.0990	6.505
	Functional Observer without Controller	0.1073	-0.0990	9.187
2	Type-2 Quasi Decentralized Functional Observer	0.0478	-0.0704	3.01
	Fuzzy Decentralized Functional Observer	0.0702	-0.0968	3.29
	Quasi Decentralized Functional Observer	0.1039	-0.1147	4.616
	Full order Luenberger Observer	0.1163	-0.1147	6.125
	Functional Observer without Controller	0.1214	-0.1147	6.967
3	Type-2 Quasi Decentralized Functional Observer	0.0487	-0.07038	2.950
	Fuzzy Decentralized Functional Observer	0.0693	-0.0943	3.24
	Quasi Decentralized Functional Observer	0.1025	-0.1108	4.804
	Full order Luenberger Observer	0.1143	-0.1108	6.326
	Functional Observer without Controller	0.1188	-0.1108	7.241
4	Type-2 Quasi Decentralized Functional Observer	0.04818	-0.07052	3.019
	Fuzzy Decentralized Functional Observer	0.06941	-0.09437	3.22
	Quasi Decentralized Functional Observer	0.1026	-0.1107	4.735
	Full order Luenberger Observer	0.1144	-0.1107	6.773
	Functional Observer without Controller	0.1189	-0.1107	7.312
5	Type-2 Quasi Decentralized Functional Observer	0.0509	-0.0676	4.14
	Fuzzy Decentralized Functional Observer	0.0636	-0.08113	4.40
	Quasi Decentralized Functional Observer	0.0851	-0.0909	6.808
	Full order Luenberger Observer	0.0917	-0.0909	6.98
	Functional Observer without Controller	0.0956	-0.0909	>10
6	Type-2 Quasi Decentralized Functional Observer	0.0487	-0.07032	3.396
	Fuzzy Decentralized Functional Observer	0.07032	-0.09322	3.726
	Quasi Decentralized Functional Observer	0.1008	-0.1073	4.857
	Full order Luenberger Observer	0.1119	-0.1073	6.407
	Functional Observer without Controller	0.1162	-0.1073	7.214
7	Type-2 Quasi Decentralized Functional Observer	0.05057	-0.06799	3.896
	Fuzzy Decentralized Functional Observer	0.06447	-0.08239	4.012
	Quasi Decentralized Functional Observer	0.08722	-0.09273	6.542
	Full order Luenberger Observer	0.09461	-0.09273	7.174
	Functional Observer without Controller	0.098938	-0.09273	> 1

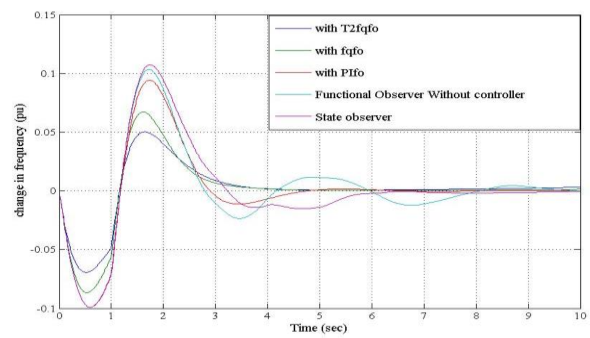

Fig. 6. Change in Frequency with Step Increase in Demand at Operating Point 1.

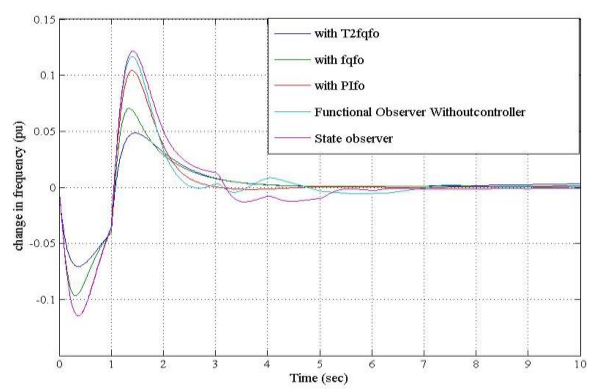

Fig. 7. Change in Frequency with Step Increase in Demand at Operating Point 2.

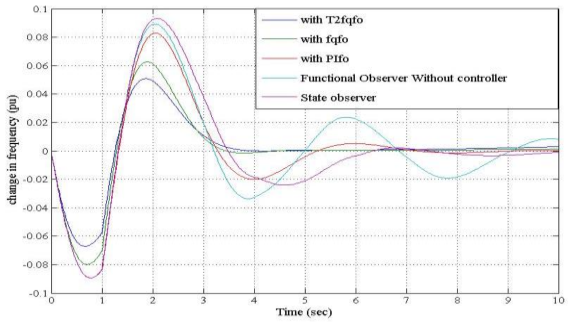

Fig. 8. Change in Frequency with Step Increase in Demand at Operating Point 3.

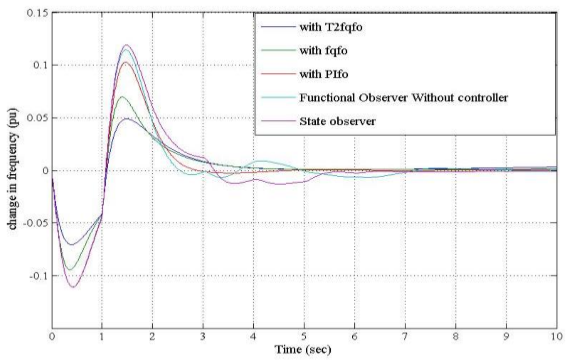

Fig. 9. Change in Frequency with Step Increase in Demand at Operating Point 4.

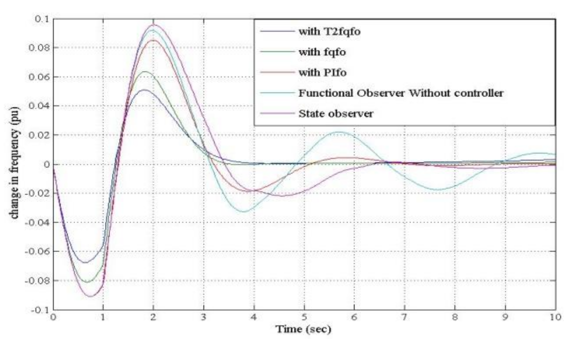

Fig. 10. Change in Frequency with Step Increase in Demand at Operating Point 5.

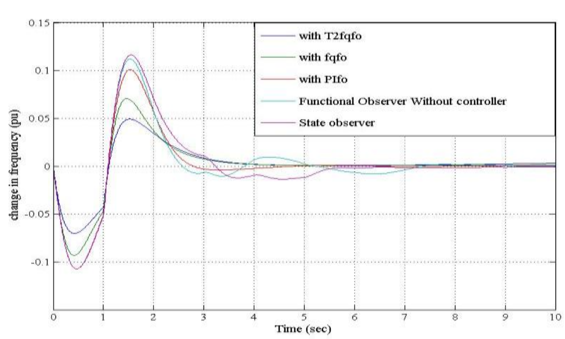

Fig. 11. Change in Frequency with Step Increase in Demand at Operating Point 6.

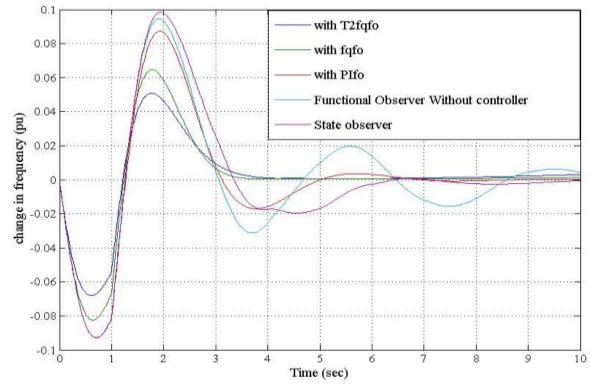

Fig. 12. Change in Frequency with Step Increase in Demand at Operating Point 7.

IV. CONCLUSIONS

In this present paper a novel Type-2 Fuzzy Functional Observer (T2FQFO) as a solution to the as a solution to the problem of load frequency control is proposed and applied to multi-area power system. Testing for disturbance attenuation and precise reference frequency tracking under different load (operating) conditions has been carried out over a typical two area interconnected reheat thermal power system with parameter uncertainties of a wide range. Competence of the proposed observer model is tested by performance comparison with FQFO based control, Functional Observer (With and without conventional controller) and Luenberger observer pertaining to settling time, maximum overshoots/undershoots under a variety of operating situations. Simulation results obtained show robustness pertaining to stability and consistency of performance of the suggested observer modeled herewith. The work may be extended by designing an optimization based design methodology using Genetic Algorithm-Fuzzy controllers or Honey Bee Mating optimization algorithm for tuning Type-2 Fuzzy PI/PID controllers. The LFC design can also be done using certain techniques like Active Disturbance rejection control for Type-2 Fuzzy system. The work may be extended with drawing extra degree of freedom framing as Type-3 Fuzzy. The relationship between the appropriate FOUs for a Type-2 FLC and the uncertainties in the plant parameters is still unsolved. If it is solved, the applications of Type-2 FLCs will be greatly promoted.

ACKNOWLEDGMENT

The authors would like to express their appreciation and gratitude to the University Management , Andhra University for granting this research study under Research Promotion Scheme(RPS) Sponsored by AICTE with File No.8-194/RIFD/RPS(POLICY-1)/2018-19.

REFERENCES

[1] M. Darouach, “Full Order Observers for Linear Systems with Unknown Inputs”, IEEE Trans. Autom. Control, vol. 39, no. 3, pp. 606–609, Mar. 1994.

[2] H.Beides and G.Heydt, “Dynamic state estimation of power system harmonics using Kalman filter methodology,” IEEE Transaction on Power Delivery,vol.6, no.4, pp.1663-1670,1991.

[3] Hieu Trinh, Tyrone Fernand, "QuasiDecentralized Functional Observers for the LFC of Interconnected Power Systems", IEEE Transactions on Power Systems, 28, no. 3, Aug. 2013.

[4] H. Trinh, T. Fernand, "Functional Observers for Dynamical Systems", Berlin, Germany: Springer Verlag, 2012.

[5] K. R. Sudha, R. Vijaya Santhi, "Robust Decentralized Load Frequency Control of Interconnected Power System with Generation Rate Constraint using Type-2 Fuzzy Approach", Science Direct Electrical Power and Energy Systems, 33, pp 699–707, 2011.

[6] L.Zhao and A.Abur, "Multi area state estimation using synchronized phasor measurement," IEEE Transaction on power system, vol.20, no.2, pp. 611-617,2005.

[7] K.Shih and S.Huang, "Application of a robust algorithm for dynamic state estimation of a power system," vol.17, no.1, pp.141-147, 2002.

[8] E.Farantatos, G.K.Stefopoulos, G.J.Cokkinides and A.Meliopoulous, "PMU Based dynamic state estimation for electric power system," in IEEE power and energy society general meeting,2009,pp.1-8.

[9] Chuang Liu, Hak-Keung Lam, Tyrone Fernando, and Herbert Ho-Ching Iu, "Design of Fuzzy Functional Observer-Controller via Higher Order Derivatives of Lyapunov Function for Nonlinear Systems," IEEE Transactions on Cybernetics, vol.pp, Issue: 99, pp.1-11,2016.

[10] M. Aldeen, J. Marsh, "DecentralisedObserver based Control Scheme for Interconnected Dynamical Systems with Unknown Inputs", IEEE Proceedings, Part D, Control Theory and Applications 146,pp.349–358, 1999.

[11] M. Aldeen, H. Trinh, "Load Frequency Control of Interconnected Power Systems via Constrained Feedback Control Schemes", Computers and Electrical Engineering 20, pp.71–88, 1994.

[12] Mendel JM, "Advances in Type-2 Fuzzy Sets and Systems", Inform Science 177, pp.84–110, 2007.

[13] M. Darouach and M. Boutayeb, Design of observers for descriptor systems,IEEE Transactions on Automatic Control, vol. 40, pp. 1323-1327, 1995.

[14] Michal Polanský, and Cemal Ardil" Robust Fuzzy Observer Design for Nonlinear Systems" International Journal of Computer, Electrical, Automation, Control and Information Engineering Vol:5, No:5, 2011.

[15] X. J. Ma, Z. Q. Sun, "Analysis and design of fuzzy controller and Fuzzy observer" IEEE Trans. Fuzzy Systems. 1998, vol. 6, no. 1, pp. 41–51..

[16] KR Sudha, RV Santhi "Load frequency control of an interconnected reheat thermal system using type-2 fuzzy system including SMES units", International Journal of Electrical Power and Energy Systems 43 (1), 1383-1392, 2012.

[17] KR Sudha, YB Raju, AC Sekhar "Fuzzy C-Means clustering for robust decentralized load frequency control of interconnected power system with Generation Rate Constraint" International Journal of Electrical Power and Energy Systems 37 (1), 58-66, 2012.

[18] R Vijaya Santhi, KR Sudha, S Prameela Devi ,"Robust Load Frequency Control of multi-area interconnected system including SMES units using Type-2 Fuzzy controller, Fuzzy Systems (FUZZY), 2013 IEEE International Conference on, pp 1-7,2013.

[19] Fang Chen, Xueshan Han,Zhiyuan Pan,Li Han "State Estimation Model and AlgorithmIncluding PMU" DRPT2008 6-9 April 2008 Nanjing China.

[20] Tyrone Fernando, Kianoush Emami, Shenglong Yu, Herbert Ho-Ching Iu, and Kit Po Wong, Fel, "A Novel Quasi-Decentralized Functional Observer Approach to LFC of Interconnected Power Systems",IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 31, NO. 4, JULY 2016.

[21] D G Luenberger, "Observer for multivariable system," IEEE Transactions on Automatic control, Vol.11, no.2, pp.190-197, 1966.