

# Design of a Multiloop Controller for a Nonlinear Process

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**Abstract**—Among the category of nonlinear processes, the Continuous Stirred Tank Reactor (CSTR) is one popular unit that finds application in various verticals of chemical process industries. The process variables within the CSTR are highly interactive; hence developing control strategies become a laborious task as it can be viewed as a Multi Input Multi Output (MIMO) system. Often the CSTR is assumed as a Single Input Single Output (SISO) system and during the development of control strategies or algorithm, the main objective is on maintaining only a single process variable closer to its set point, even though many measured variables form part of it. On the contrary, when compared to a SISO system, the MIMO control includes sustaining different controlled variables at their appropriate set points concurrently; thereby achieving an improved efficiency. The components' concentration and the temperature inside the CSTR are highly interactive in nature and exhibit reasonably high non-linear steady state behaviour. Both the interaction and non-linear behaviours pose challenges to the overall system stability. A stabilizing Proportional + Integral (PI) controller employing Stability Boundary Locus (SBL) concept is designed for a CSTR which eventually encapsulates both the stability and closed loop performance in its design procedure and analysed through simulation in MATLAB with the results presented.

**Keywords**—Nonlinear process; interaction; multi input multi output control; closed loop performance; stability

## I. INTRODUCTION

A process industry consists of various process units coupled together to perform a process operation and hence they are typically Multi Input and Multi Output (MIMO) systems. However, to implement process control strategies, process units are treated as Single Input Single Output (SISO) systems [1,2]. The most important notion is in maintaining a particular process variable closer to the set point, although quite a few measured variables are part of it [3]. Generally, the interactions amongst those variables are not considered in the control system design. This results in the augmented use of energy and therefore upsurges the plant's operational costs. In contrast to SISO, the MIMO control objective performs to maintain quite a lot of controlled variables at their desired set points at the same time. For the control of MIMO systems, design technique

of single loop tuned controller can't be implemented directly due to the heavy interaction between the loops that inflict intricacies in the control system design. For a MIMO system two kinds of control system design exists. The first type is the multivariable control method, where a solitary control algorithm oversees the control of all interacting loops within the process therefore the fail-safe design becomes more complicated [4]. The second type is the Multiloop control, a kind of multiloop control of individual loop that demonstrates a natural immunity to the loop failure thereby resulting in an easy and potent fail-safe design. The interaction within the CSTR is phenomenal with the Biggest Log modulus Tuning (BLT) [5,6] been implemented earlier [7] and in this present work, the performance of the same CSTR system is analyzed by employing Stability Boundary Locus (SBL) concept through simulation in MATLAB. The SBL concept graphically defines a boundary in the parametric design plane of the controller, to separate the stable and unstable regions of a feedback control system. The value addition of this paper lies in implementing the proposed stabilizing PI controller employing the Stability Boundary Locus (SBL) concept to specify the choice of controller parameters that results in the stable operation of the chosen highly non-linear CSTR process.

The paper is structured as below. In the subsequent sections, related work and then the equations related to the CSTR model and parameters are provided, followed by the multiloop control scheme. The next section details the design steps involved in the MIMO systems to plot the stability boundary loci to calculate stabilizing PI controllers for all the varied operating points of the CSTR. Simulation results to validate the control performance of the proposed method followed by discussion and conclusion are provided afterwards.

## II. RELATED WORK

Nusret Tan et al. (2006) have proposed a method to compute all the parameters of a PI controller which stabilize a control system [8]. Hanwate and Hote (2014) have designed a PID controller for cart inverted pendulum system based on the concept of stability boundary locus [9]. A mathematical model of the DC motor control system has been derived by Praboo

and Bhaba (2014) based on the model fractional order PI{\lambda} controller using the stability boundary locus method to satisfy the required gain margin (GM) and phase margin (PM) of the system [10]. The work by Deniz et al. (2016) has introduced an integer order approximation method for the numerical implementation of fractional order derivative/integrator operators in control system based on fitting the stability boundary locus (SBL) of fractional order derivative/integrator operators and SBL of integer order transfer functions [11]. A generalized approach to identify all stabilizing PI controllers for processes with time delay that depends on modeling higher order plant transfer functions by a first order plus dead time model has been proposed by Kaya and Atiç (2016) from which the normalized form of the obtained model and controller transfer functions were used for plotting the stability boundary locus plane [12]. The computation of all stabilizing PI controllers for third order systems obtained using the Boundary locus and Kronecker summation method to guarantee the stability of a feedback system was proposed by Amarendra Reddy et al. (2017) [13]. The paper by Atic and Kaya. (2018) has proposed a method by which all PID controller tuning parameters, satisfying stability of any unstable time delay processes, can be calculated by forming the stability boundary loci [14]. A method based on stability region locus and the Mikhailov criterion for stability test has been proposed to determine the parameters of PI controllers to control a TITO (two-input two-output) NCS (networked control systems) with intrinsic and network induced time delays has been proposed by Mohamed-Vall (2021) [15].

### III. CSTR MODEL AND PARAMETERS

An irreversible first order exothermic reaction (A→B) occurring in a CSTR as presented in Fig. 1 is considered. A cooling jacket that surrounds the reactor get rid of the heat produced during the reaction. It is assumed that perfect mixing occurs within the CSTR and also any changes in the volume owing to the reaction are considered negligible. As per the above assumptions, jacket water tends to be perfectly mixed with a continual water hold up happening within the jacket and the weight of the CSTR metal walls also being regarded negligible [16]. The fundamental model along with the resultant operating points of the CSTR is provided in Table I [17, 18] and the same has been taken up for the simulation studies.

The dynamics of process variables in the CSTR is given by;

$$\frac{dC_A}{dt} = \frac{F}{V}C_{A0} - \frac{F}{V}C_A - C_A K_0 e^{(-E/RT)} \quad (1)$$

$$\frac{dT}{dt} = \frac{F}{V}T_{in} - \frac{F}{V}T - \frac{H_r C_A K_0 e^{(-\frac{E}{RT})}}{\rho C_p} - \frac{UA}{\rho C_p V} (T - T_c) \quad (2)$$

$$\frac{dT_c}{dt} = \frac{F_c}{V_c} (T_{cin} - T_c) + \frac{UA}{\rho_c V_c C_{pc}} (T - T_c) \quad (3)$$

From the modelling equations of CSTR, it is evident that the process variables  $C_A$ ,  $T$  and  $T_c$  remain to be a nonlinear function. Also due to their interactive nature, they cannot be determined independently. The Table I exhibit the CSTR's steady state operational considerations taken up herein.

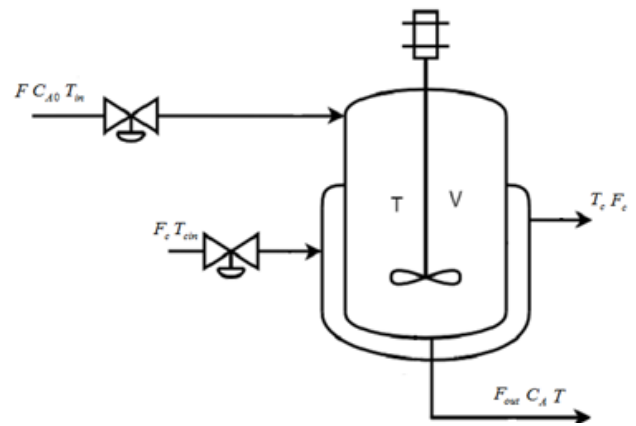


Fig. 1. Continuous Stirred Tank Reactor.

TABLE I. NOMINAL CSTR PARAMETERS AND VARIABLES

| Variable  | Description   | Nominal Operating Values |
|-----------|---|--------------------------|
| V         | Reactor volume (l)                                      | 50                       |
| $F_{in}$  | Reactor's inlet volumetric flow rate (l/min)            | 50                       |
| $F_{out}$ | Reactor's outlet volumetric flow rate (l/min)           | 50                       |
| $C_A$     | Component A concentration in the outlet stream (mole/l) | -                        |
| $C_{A0}$  | Feed concentration of component A (mole/l)              | 1                        |
| $K_0$     | Pre-exponential factor (l/min)                          | $7.8 * 10^{10}$          |
| E         | Activation energy in the Arrhenius equation (Cal/mole)  | E/R=8567                 |
| R         | Universal gas constant (Cal/mole. K)                    |                          |
| $\rho$    | Density of inlet and outlet stream (g/l)                | 900                      |
| $C_p$     | Inlet and Outlet streams - Heat capacity (Cal/g.K)      | 0.329                    |
| T         | Reactants temperature in the reactor (K)                | -                        |
| $T_{in}$  | Temperature of the Inlet stream (K)                     | 350                      |
| $H_r$     | Reaction's Heat (Cal/mole)                              | $-5*10^4$                |
| UA        | Heat transfer term (Cal/min. K)                         | $5*10^4$                 |
| $T_c$     | Jacket's coolant water Temperature (K)                  | -                        |
| $\rho_c$  | Jacket's coolant water density (g/l)                    | 1000                     |

### IV. MULTILoop CONTROL OF CSTR

A CSTR is a specialized MIMO system that contains two controlled variables (CV), reactor concentration and temperature. These controlled variables have to be maintained at their nominal operating values. Relative Gain Array (RGA) analysis is carried out on the considered CSTR model to recommend the best pairing [19].

Based on the loop pairing, it is proposed that, to attain the best closed loop performance, the flowrate of the inlet coolant water  $F_c$  ought to be paired with the concentration of Component A in the outlet stream  $C_A$  and the inlet flowrate  $F$

need to be paired with the reactor temperature T. The multiloop control structure engaged in this effort is given in Fig. 2.

The RGA analysis is carried out for the proposed CSTR model and is calculated as given in (4).

$$\lambda = \frac{F_c}{F} \begin{bmatrix} 3.9880 & -2.9880 \\ -2.9880 & 3.9880 \end{bmatrix} \quad (4)$$

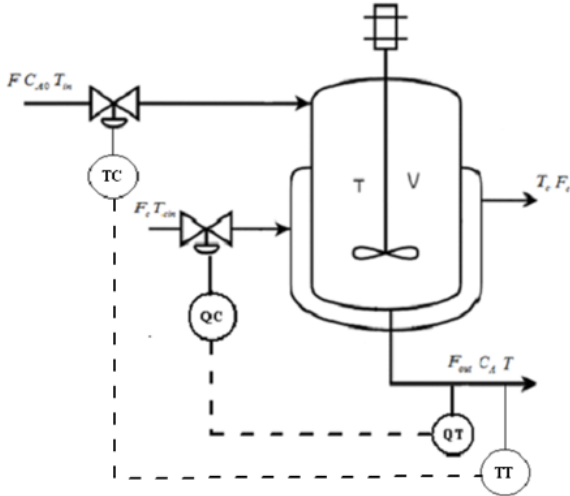


Fig. 2. Multiloop Control Scheme for CSTR.

The RGA analysis specifies that  $F_C$  needs to be paired with  $C_A$ , and  $F$  paired along with  $T$  to offer superior control. As  $\lambda_{ij}$  is seen as a small positive value, the gain in closed-loop is considerably greater than in open-loop. This phenomenon may be a source for decline in performance or instability when the loop is closed. This accentuates that, an input in open-loop that possess small influence on a specific output will hold a noteworthy effect in closed-loop owing to feedback as well as coupling.

#### V. DESIGN OF MULTILoop STABILIZING PI CONTROLLER FOR CSTR

The robust performance and simplicity of the stabilizing PI (Proportional + Integral), PID (Proportional + Integral + Derivative) and lag/lead design find themselves being extensively used in the process industries. The Stability Boundary Locus (SBL) method performs the computation of the PI controller parameters which stabilizes the control system proposed by [20,21]. A stability boundary locus in the  $(K_C, K_I)$  plane is plotted and the stabilizing values of P+I controller parameters are evaluated. This method does not necessitate sweeping over the parameters and also does not require linear programming to solve a set of inequalities. The stabilizing PI controller evolved from the stability boundary locus not only guarantees the stability but also the desired closed-loop transients are obtained [8]. The following sections explain briefly the design steps involved in the MIMO systems.

##### A. Stabilizing PI Controller for First Order Plus Dead Time (FOPDT) Process

For a SISO system with,

$$G_p(s) = G(s)e^{-\theta s} = \frac{N(s)}{D(s)}e^{-\theta s} \quad (5)$$

and a P+I controller of the form

$$G_c(s) = K_c + \frac{K_i}{s} \quad (6)$$

where the problem is to compute the PI controller parameters that stabilizes the specified system [22,23]. The system's closed loop characteristic polynomial  $P(s)$  [24] is;

$$P(s) = sD(s) + (K_c s + K_i)N(s)e^{-\theta s} \\ = a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0 \quad (7)$$

It is to be noted that all coefficients or some of the coefficients  $a_i, i = 0, 1, 2, \dots, n$  are the function of  $K_C, K_I$  and  $e^{-\theta s}$  depends on the order of polynomials  $N(s)$  and  $D(s)$ . There exist 3 eventualities for a stable polynomial's root to cross over the imaginary axes, (i.e., the polynomial turns out to be unstable) in the parameter space approach. (i) Real Root Boundary: At  $s = 0$ , a real root crosses over the imaginary axis. As a result, the real root boundary can be attained from  $P(s)$  in (7) by substituting  $s = 0$  which provides  $a_0 = 0$ . (ii) Infinite Root Boundary: At  $s = \infty$ , a real root crosses over the imaginary axis. Therefore, the infinite root boundary can be denoted by considering  $a_n = 0$  from (7) (iii) Complex Root Boundary: The polynomial typified by (7) becomes unstable at  $s = j\omega$  while the roots cross the imaginary axis that signifies the real and the imaginary parts of (7) all becoming zero at the same time. Hence the complex root boundary can be attained as given below. The numerator and denominator polynomials of  $G_p(s)$  can be decomposed into their equivalent odd and even parts by replacing  $s$  with  $j\omega$  which yields.

$$G(j\omega) = \frac{N_e(-\omega^2) + j\omega N_o(-\omega^2)}{D_e(-\omega^2) + j\omega D_o(-\omega^2)} \quad (8)$$

For the sake of simplicity  $(-\omega^2)$  will not be part of the ensuing equations. The closed loop characteristic polynomial of (7) can be written as;

$$\Delta(j\omega) = [(K_i N_e - K_c \omega^2 N_o) \cos(\omega\theta) + \omega(K_i N_o + K_c N_e) \sin(\omega\theta) - \omega^2 D_o] \\ + j[\omega(K_i N_o + K_c N_e) \cos(\omega\theta) - (K_i N_e - K_c \omega^2 N_o) \sin(\omega\theta) + \omega D_e] \\ = R_\Delta + jI_\Delta = 0 \quad (9)$$

Upon equating the real and imaginary parts of  $\Delta(j\omega)$  to zero;

$$K_c[-\omega^2 N_o \cos(\omega\theta) + \omega N_e \sin(\omega\theta)] + \\ K_i[N_e \cos(\omega\theta) + \omega N_o \sin(\omega\theta)] = \omega^2 D_o \quad (10)$$

$$K_c[\omega N_e \cos(\omega\theta) + \omega^2 N_o \sin(\omega\theta)] + \\ K_i[\omega N_o \cos(\omega\theta) - N_e \sin(\omega\theta)] = -\omega D_e \quad (11)$$

Splitting into

$$Q(\omega) = \omega N_e \sin(\omega\theta) - \omega^2 N_o \cos(\omega\theta) \\ R(\omega) = \sin N_e \cos(\omega\theta) + \omega N_o \sin(\omega\theta) \\ X(\omega) = \omega^2 D_o \quad (12)$$

$$S(\omega) = \omega N_e \cos(\omega\theta) + \omega^2 N_o \sin(\omega\theta)$$

$$U(\omega) = \omega N_o \cos(\omega\theta) - N_e \sin(\omega\theta)$$

$$Y(\omega) = -\omega D_e \quad (13)$$

Then (10) and (11) can be denoted as

$$K_c Q(\omega) + K_i R(\omega) = X(\omega) \quad (14)$$

$$K_c S(\omega) + K_i U(\omega) = Y(\omega) \quad (15)$$

Solving (14) and (15)

$$K_c = \frac{X(\omega)U(\omega) - Y(\omega)R(\omega)}{Q(\omega)U(\omega) - R(\omega)S(\omega)} \quad (16)$$

and

$$K_i = \frac{Y(\omega)Q(\omega) - X(\omega)S(\omega)}{Q(\omega)U(\omega) - R(\omega)S(\omega)} \quad (17)$$

Upon substituting (12) and (13) into (16) and (17), it is found that,

$$K_c = \frac{(\omega^2 N_0 D_0 + N_e D_e) \cos(\omega\Theta) + \omega(N_0 D_e - N_e D_0) \sin(\omega\Theta)}{-(N_e^2 + \omega^2 N_0^2)} \quad (18)$$

$$K_i = \frac{\omega^2(N_0 D_e - N_e D_0) \cos(\omega\Theta) - \omega(N_e D_e + \omega^2 N_0 D_0) \sin(\omega\Theta)}{-(N_e^2 + \omega^2 N_0^2)} \quad (19)$$

It can be perceived that if the denominator of (18) and (19)  $N_e(-\omega^2) + \omega^2 N_0(-\omega^2) \neq 0$ , then the stability boundary locus,  $l(K_c, K_i, \omega)$  can be constructed in the  $(K_c, K_i)$ -plane. Whereas if at any specific value of frequency, the denominator of equations (18) and (19)  $N_e(-\omega^2) + \omega^2 N_0(-\omega^2) = 0$ , then it implies that the frequency value should not be used. Here a discontinuous stability boundary locus will be attained which won't be problematic as far as the stabilizing controller's computation is concerned. When the stability boundary locus is attained, it becomes essential to assess if the stabilizing controllers are existent or not. This is because the stability boundary locus, the real root and infinite root boundary lines may perhaps split the parameter plane into stable and unstable regions. It can be seen that the line  $K_i = 0$  can split the parameter plane  $(K_c, K_i)$  into two regions viz. stable and unstable. In this case the line  $K_i = 0$  is the real root boundary line attained by substituting  $\omega = 0$  in (7) and then equating it to zero as a real root of  $\Delta(s)$  of (7) may cross over the imaginary axis at  $s = 0$ .

Generally, for a transfer function, the order of  $D(s)$  is greater than the order of  $N(s)$  which guarantees no infinite root boundary line. The stability boundary locus is seen to be reliant on the frequency  $\omega$  that varies from 0 to  $\infty$  which signifies the importance of frequency gridding. The lessening of the range of frequencies that desires to be gridded can be effectively achieved by employing the Nyquist plot based method as provided in [25]. Here we need to find only the real values of  $\omega$  that satisfy  $\text{Im}[G(s)] = 0$  where  $s = j\omega$ . As the controller operates in this frequency range, it indicates that the frequency below the critical frequency  $\omega_c$  or the ultimate frequency can be considered. For that reason, in order to get the stability boundary locus over a likely smaller range of frequency such as  $\omega \in [0, \omega_c]$ , the critical frequency can be employed. As the phase of  $G_p(s)$  at  $s = j\omega_c$  is equal to  $-180^\circ$ , it can be written as;

$$\tan^{-1} \left( \frac{\omega N_0}{N_e} \right) - \tan^{-1} \left( \frac{\omega D_0}{D_e} \right) - \omega\theta = -\pi \quad (20)$$

$$\text{or } \tan(\omega\theta) = \frac{\omega(N_0 D_e - N_e D_0)}{N_e D_e + \omega^2 N_0 D_0} = f(\omega) \quad (21)$$

### B. Implementation of SBL Algorithm for CSTR Control

From the models obtained in the resultant three operating regions of the CSTR, the stability boundary loci for the loops are obtained using (16) and (17). The boundary loci are depicted in Fig. 3. The  $(K_c, K_i)$  points are obtained by varying  $\omega$  from 0 to  $\omega_c$  in steps of 0.001 accordingly. The Multiloop controller parameters are obtained from stability boundary locus [26] using weighted geometrical centre with the aid of (22) and (23).

$$K_c = \frac{1}{n} \sum_{j=1}^n K_{cj} \quad (22)$$

$$K_i = \frac{1}{2n} \sum_{j=1}^n K_{ij} \quad (23)$$

The steady-state profile has yielded the notion of operating the CSTR at three diverse operating regions. In order to investigate the multiloop control of the CSTR, the operating points are prudently selected as per the steady-state input-output response as low ( $F = 70$  l/min,  $F_C = 60$  l/min), middle ( $F = 31$  l/min,  $F_C = 99$  l/min) and high ( $F = 25$  l/min,  $F_C = 115$  l/min).

The steady-state value of the three chosen operating regions is presented in Table II.

TABLE II. OPERATING POINTS OF CSTR-MIMO PROCESS

| Operating Region | $C_A$ (mole/l) | T (K) | $T_C$ (K) |
|------------------|----------------|-------|-----------|
| Low              | 0.05725        | 390.4 | 341.1     |
| Middle           | 0.7963         | 318.3 | 306.1     |
| High             | 0.8178         | 314.1 | 304.3     |

The MIMO model for the chosen three operating regions (low, middle and high) obtained using the process reaction curve (PRC) method are specified from (24) to (26) respectively.

$$G(s) = \begin{bmatrix} \frac{0.84e^{-0.1s}}{0.3s+1} & \frac{-0.46e^{-0.15s}}{0.75s+1} \\ \frac{-0.0017e^{-0.15s}}{0.45s+1} & \frac{0.0015e^{-0.25s}}{0.75s+1} \end{bmatrix} \quad (24)$$

$$G(s) = \begin{bmatrix} \frac{0.5e^{-0.1s}}{1.2s+1} & \frac{-0.2e^{-0.05s}}{1.35s+1} \\ \frac{-0.0025e^{-2.05s}}{1.35s+1} & \frac{0.0039e^{-1.35s}}{1.65s+1} \end{bmatrix} \quad (25)$$

$$G(s) = \begin{bmatrix} \frac{0.48e^{-0.1s}}{0.9s+1} & \frac{-0.04e^{-0.4s}}{0.6s+1} \\ \frac{-0.00068e^{-3.3s}}{1.2s+1} & \frac{0.00056e^{-0.9s}}{1.8s+1} \end{bmatrix} \quad (26)$$

For the locally linearized models, the Multiloop controllers are designed at the chosen three operating points by employing the SBL technique. For the control scheme, the feed flow rate ( $F$ ) and coolant flow rate ( $F_C$ ) remain the manipulated variables (MV) whereas the concentration of component A ( $C_A$ ) and reactor temperature ( $T$ ) are the controlled variables. The design stage involves finding the PI controller parameters initially by means of the Ziegler Nichols method [27] for individual loop, with the multiloop controller parameters

adjusted thereupon by making use of the detuning factor  $f$ . For this CSTR process, the detuning factor 'f' for the chosen three operating regions is estimated to be 1.195, 1.275 and 1.26 correspondingly.

### C. Control Performance of Multiloop Stability Boundary Locus (SBL) method – Reactor Concentration Control

To investigate the closed loop system behaviour with the controller so designed, the concentration setpoints are changed over all the three operating regions. The Feed concentration  $C_{A0}$ , the temperature of the Inlet stream  $T_{in}$  and the inlet coolant water temperature in the jacket  $T_{cin}$  act as the disturbances for this system, which are intentionally changed from the nominal values at various sampling instants. All the figures in this and subsequent sections consist of two halves. The top half shows the trends of the process variables (PV) for the changes in the setpoints and disturbances. The bottom half

displays the corresponding changes in the manipulated variable which drives the PV to the setpoints. Fig. 4, 6 and 8 show the servo and regulatory responses for the operated low, middle and high regions.

It is found that the reactor concentration tracks the setpoint changes without any offset and at the same time the disturbance rejection is also phenomenal. The three different disturbances in the CSTR mentioned earlier are introduced intentionally at the sampling instances of 40, 80 and 190 respectively. The rejection trend shows how effectively the disturbances are removed. Fig. 5, 7 and 9 show the interaction effect of concentration on the reactor temperature. In all the above-mentioned figures it is found that the process being MIMO in nature, remains susceptible to the interaction effect. However, the degree of interaction is very much reduced due to the effective SBL design methodology.

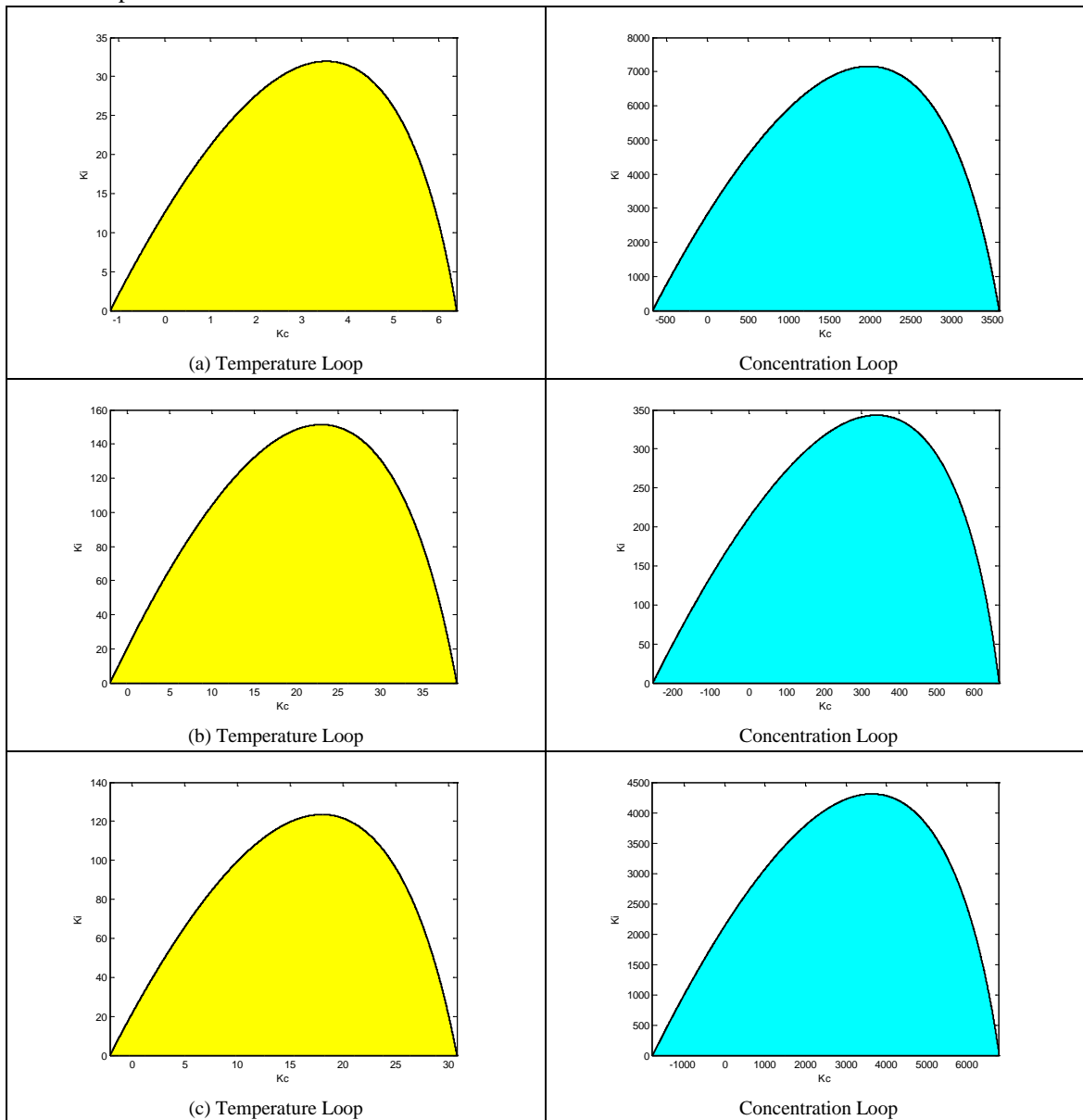


Fig. 3. Stability Regions of the CSTR Process (a) Low (b) Middle (c) High.

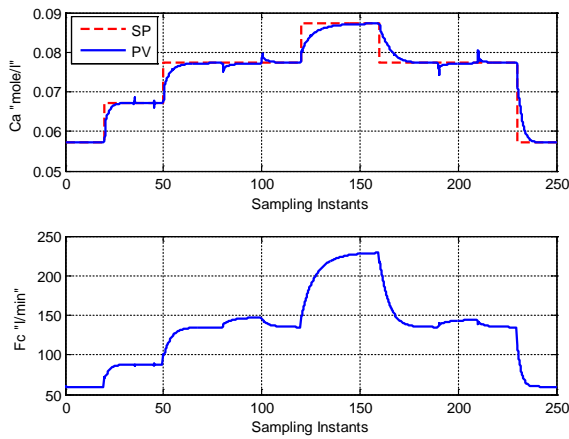


Fig. 4. Servo and Regulatory Responses of Multiloop SBL Design for Concentration Control at Low Region.

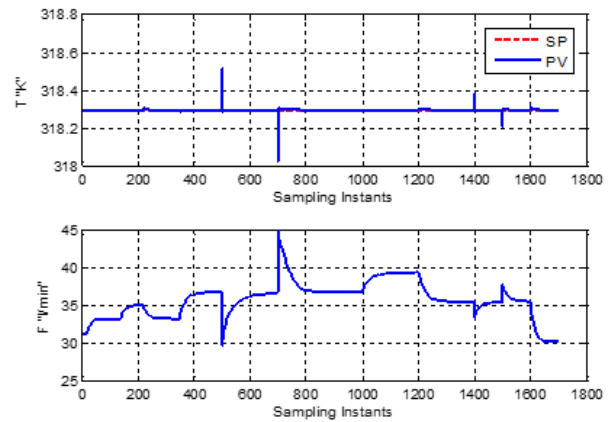


Fig. 7. Interaction Behaviour of Temperature with SBL Design for Concentration Control at Middle Region.

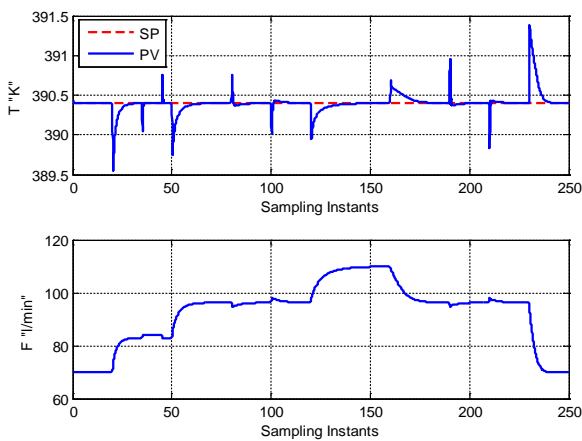


Fig. 5. Interaction Behaviour of Temperature with SBL Design for Concentration Control at Low Region.

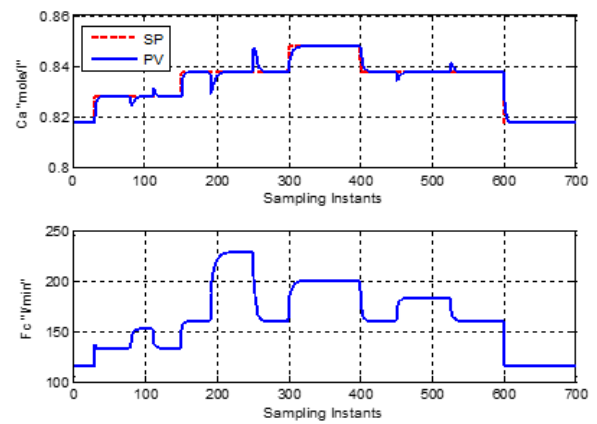


Fig. 8. Servo and Regulatory Responses of Multiloop SBL Design for Concentration Control at High Region.

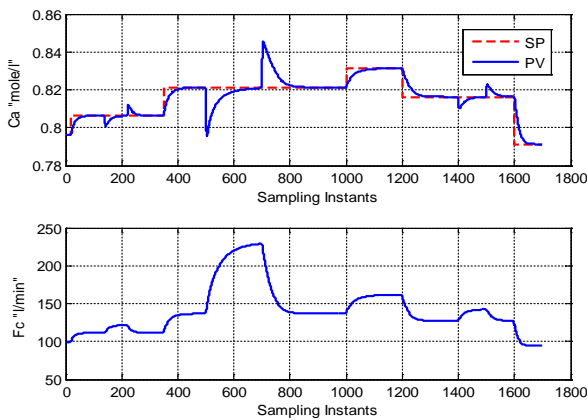


Fig. 6. Servo and Regulatory Responses of Multiloop SBL Design for Concentration Control at Middle Region.

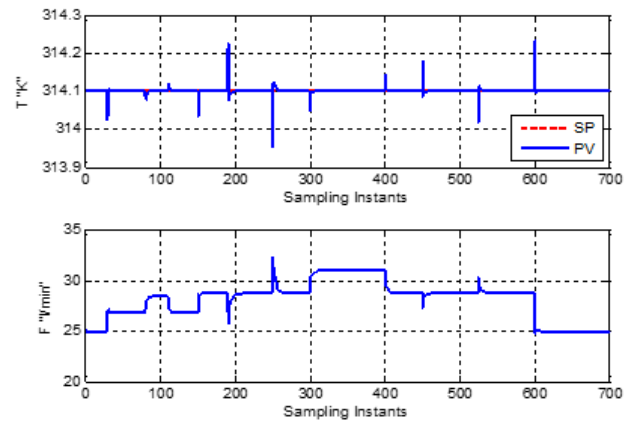


Fig. 9. Interaction Behaviour of Temperature with SBL Design for Concentration Control at High Region.



#### D. Control Performance of Multiloop Stability Boundary Locus (SBL) Method - Reactor Temperature Control

The closed-loop performances of the designed controller are studied in all three chosen operating regions of the CSTR. The servo and regulatory responses are obtained for the analogous setpoint and disturbance patterns employed already for concentration control. Fig. 10, 12 and 14 show the servo and regulatory responses. The step change in the setpoints is made at various sampling instances in both directions to check the setpoint tracking feature of the designed controller. It is found that the designed controller behaves well.

The bottom halves of the figure show the trend of manipulated variable (the feed flow rate). It is also observed that the SBL method smoothly rejects the interaction effect on concentration due to changes in temperature setpoints and disturbances which are recorded in Fig. 11, 13 and 15.

The spikes in the trends on the top halves of the figures show the deviation of concentration from its constant nominal values at various sampling instances. The servo and regulatory responses are obtained for both the concentration and temperature control as per the controller tuning constants obtained using multiloop controllers based on the Stability Boundary Locus method and the same is provided in Table III.

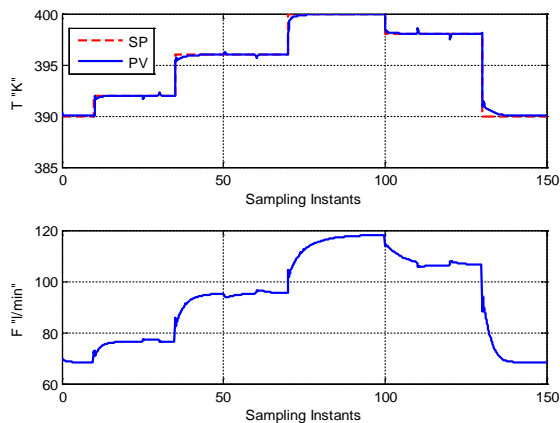


Fig. 10. Servo and Regulatory Responses of Multiloop SBL Design for Temperature Control at Low Region.

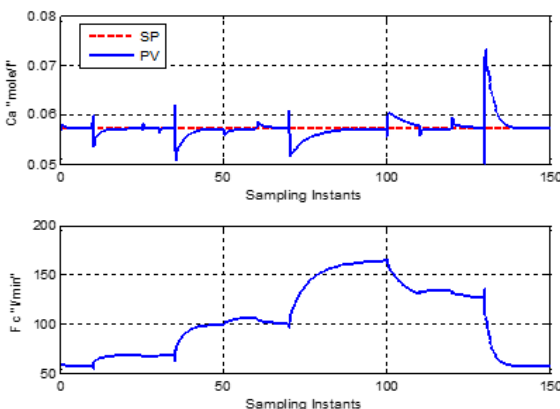


Fig. 11. Interaction Behavior of Concentration with SBL Design for Temperature Control at Low Region.

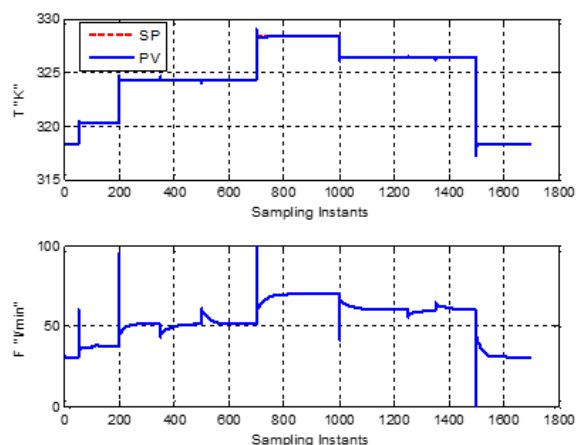


Fig. 12. Servo and Regulatory Responses of Multiloop SBL Design for Temperature Control at Middle Region.

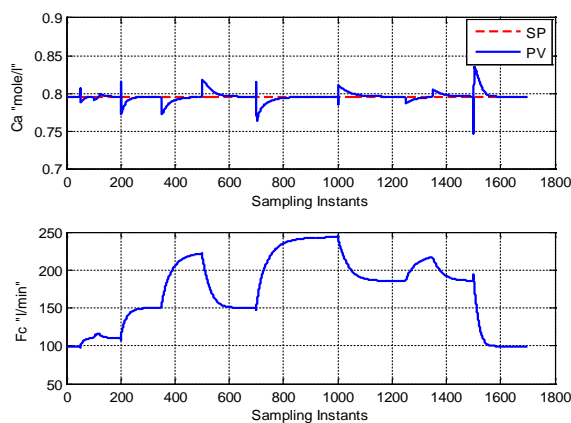


Fig. 13. Interaction Behavior of Concentration with SBL Design for Temperature Control at Middle Region.

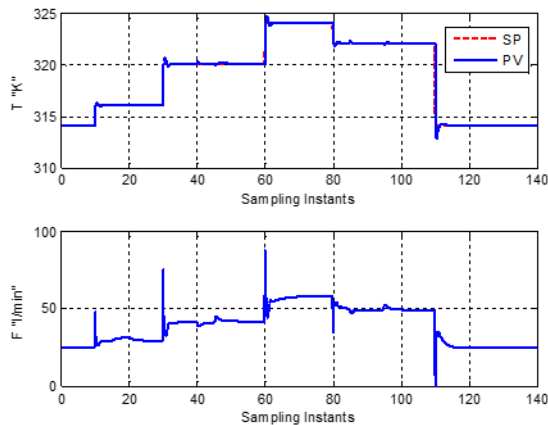


Fig. 14. Servo and Regulatory Responses of Multiloop SBL Design for Temperature Control at High Region.

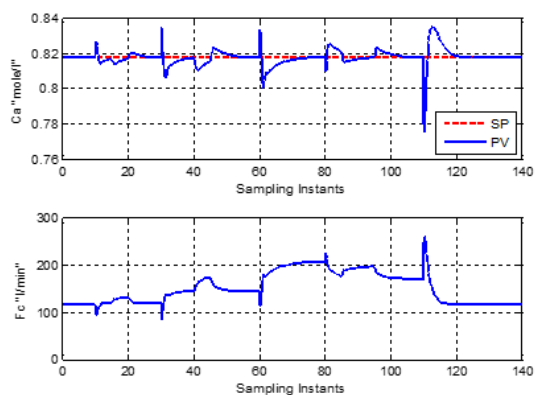


Fig. 15. Interaction Behavior of Concentration with SBL Design for Temperature Control at the High Region.

TABLE III. MULTILoop CONTROLLER PARAMETERS FOR CSTR PROCESS USING SBL DESIGN METHOD

| Operating Region | $K_c$            |                    | $K_i$            |                    |
|------------------|------------------|--------------------|------------------|--------------------|
|                  | Temperature Loop | Concentration Loop | Temperature Loop | Concentration Loop |
| Low              | 1.9942           | 1117.5             | 8.6737           | 1.942              |
| Middle           | 14.7728          | 146.9798           | 40.9988          | 93.4175            |
| High             | 11.4149          | 1875.0             | 33.4506          | 1170.5             |

TABLE IV. PERFORMANCE INDICES OF MULTILoop SBL DESIGN FOR SETPOINT CHANGE DURING CONCENTRATION CONTROL

| Operating Regions | Setpoint Change | ISE      |          | IAE      |          |
|-------------------|-----------------|----------|----------|----------|----------|
|                   |                 | $T$      | $C_A$    | $T$      | $C_A$    |
| Low               | 0.0572 - 0.0673 | 6.028386 | 0.000579 | 15.0567  | 0.149571 |
|                   | 0.0673 - 0.0772 | 4.29804  | 0.000921 | 15.41896 | 0.247092 |
|                   | 0.0772 - 0.0872 | 2.623592 | 0.001821 | 15.6639  | 0.487632 |
|                   | 0.0872 - 0.0772 | 2.050618 | 0.00241  | 15.66487 | 0.487642 |
|                   | 0.0772 - 0.0572 | 1.18E-07 | 0.0004   | 0.000303 | 0.020007 |
| Middle            | 0.7963 - 0.8063 | 0.001325 | 0.006266 | 0.497038 | 1.306073 |
|                   | 0.8063 - 0.8163 | 0.002623 | 0.018643 | 0.842773 | 2.70376  |
|                   | 0.8163 - 0.8263 | 0.00294  | 0.041567 | 1.484041 | 5.97387  |
|                   | 0.8263 - 0.8163 | 0.002003 | 0.029539 | 0.941316 | 3.686016 |
|                   | 0.8163 - 0.7963 | 0.007016 | 0.048059 | 1.272217 | 3.524355 |
| High              | 0.8178 - 0.8278 | 0.034768 | 0.000932 | 0.67775  | 0.163779 |
|                   | 0.8278 - 0.8378 | 0.025621 | 0.001183 | 0.598351 | 0.228391 |
|                   | 0.8378 - 0.8478 | 0.017796 | 0.001645 | 0.674321 | 0.343457 |
|                   | 0.8478 - 0.8378 | 0.014713 | 0.001831 | 0.674334 | 0.343457 |
|                   | 0.8378 - 0.8178 | 0.114312 | 0.004482 | 1.228779 | 0.39217  |

The servo and regulatory performances of the designed controller are analyzed by computing the ISE (Integral square error) and IAE (Integral Absolute Error) [28] at various operating points within the three operating regions of the CSTR. The performance indices are computed during concentration and temperature control and tabulated. Table IV and Table VII display the performance measures calculated for each setpoint change during concentration and temperature control respectively. The performance measures computed during disturbance changes are given in Table V and Table VI for concentration and temperature control, respectively.

TABLE V. PERFORMANCE INDICES OF MULTILoop SBL DESIGN FOR DISTURBANCE CHANGE DURING CONCENTRATION CONTROL

| Operating Regions | Disturbance Change | ISE         |             | IAE         |          |
|-------------------|--------------------|-------------|-------------|-------------|----------|
|                   |                    | $T$         | $C_A$       | $T$         | $C_A$    |
| Low               | $C_{A0}$           | 0.261004882 | 3.38187E-06 | 1.145382264 | 0.00461  |
|                   | $T_{cin}$          | 0.427337781 | 5.19373E-05 | 3.433905973 | 0.061376 |
|                   | $T_{in}$           | 0.560756951 | 3.78241E-05 | 2.727808278 | 0.045502 |
| Middle            | $C_{A0}$           | 0.000944948 | 0.003304249 | 0.437309074 | 1.082304 |
|                   | $T_{cin}$          | 0.261543941 | 0.121135311 | 3.214269128 | 9.889035 |
|                   | $T_{in}$           | 0.639564818 | 0.309675259 | 8.537920774 | 26.11381 |
| High              | $C_{A0}$           | 0.006169874 | 0.000339233 | 0.49342688  | 0.157271 |
|                   | $T_{cin}$          | 0.086943188 | 0.003011574 | 1.613168071 | 0.580593 |
|                   | $T_{in}$           | 0.023119094 | 0.000446222 | 0.70756471  | 0.201456 |

TABLE VI. PERFORMANCE INDICES OF MULTILoop SBL DESIGN FOR DISTURBANCE CHANGE DURING TEMPERATURE CONTROL

| Operating Regions | Disturbance Change | ISE         |             | IAE         |             |
|-------------------|--------------------|-------------|-------------|-------------|-------------|
|                   |                    | $T$         | $C_A$       | $T$         | $C_A$       |
| Low               | $C_{A0}$           | 0.205901436 | 1.98753E-06 | 1.013312227 | 0.003348628 |
|                   | $T_{cin}$          | 0.275668499 | 2.09492E-05 | 2.854061744 | 0.034021004 |
|                   | $T_{in}$           | 0.654673618 | 7.08659E-06 | 2.251541109 | 0.013654934 |
| Middle            | $C_{A0}$           | 0.001294486 | 0.00268816  | 0.372627445 | 0.694312277 |
|                   | $T_{cin}$          | 0.399054716 | 0.087630653 | 3.711265015 | 7.675565711 |
|                   | $T_{in}$           | 0.135731116 | 0.017026752 | 1.75823604  | 3.348912205 |
| High              | $C_{A0}$           | 0.009394668 | 0.000210094 | 0.487764198 | 0.094506423 |
|                   | $T_{cin}$          | 0.102501726 | 0.001058962 | 1.434894951 | 0.211575404 |
|                   | $T_{in}$           | 0.1104586   | 0.000212938 | 1.209813883 | 0.123608686 |



TABLE VII. PERFORMANCE INDICES OF MULTILoop SBL DESIGN FOR SETPOINT CHANGE DURING TEMPERATURE CONTROL

| Operating Regions | Setpoint Change | ISE             |                 | IAE             |                 |
|-------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                   |                 | T               | C <sub>A</sub>  | T               | C <sub>A</sub>  |
| Low               | 390 - 392       | 7.278633<br>732 | 9.19617<br>E-05 | 10.30603<br>635 | 0.059160<br>341 |
|                   | 392 - 396       | 29.30308<br>893 | 0.000482<br>27  | 23.56951<br>958 | 0.170717<br>264 |
|                   | 396 - 400       | 29.51332<br>384 | 0.000925<br>221 | 28.19125<br>551 | 0.332755<br>641 |
|                   | 400 - 398       | 7.363435<br>376 | 0.000355<br>801 | 13.07605<br>805 | 0.169436<br>652 |
|                   | 398 - 390       | 64.08801<br>105 | 8.71647<br>E-09 | 8.005466<br>429 | 9.31349<br>E-05 |
| Middle            | 318.3 - 320.3   | 4.748005<br>943 | 0.006064<br>203 | 4.636015<br>839 | 1.262702<br>397 |
|                   | 320.3 - 324.3   | 19.66531<br>355 | 0.050494<br>916 | 10.62003<br>549 | 4.398186<br>96  |
|                   | 324.3 - 328.3   | 21.24057<br>865 | 0.138593<br>034 | 13.02562<br>158 | 10.09649<br>517 |
|                   | 328.3 - 326.3   | 5.194835<br>414 | 0.050938<br>797 | 6.096147<br>516 | 6.234963<br>292 |
|                   | 326.3 - 318.3   | 64.00049<br>012 | 1.13771<br>E-08 | 8.000030<br>629 | 0.000106<br>654 |
| High              | 314.1 - 316.1   | 5.098656<br>774 | 0.000610<br>959 | 5.200968<br>794 | 0.150966<br>628 |
|                   | 316.1 - 320.1   | 21.27278<br>662 | 0.002822<br>406 | 11.40449<br>762 | 0.379213<br>217 |
|                   | 320.1 - 324.1   | 22.31137<br>176 | 0.005225<br>345 | 12.51482<br>279 | 0.641269<br>532 |
|                   | 324.1 - 322.1   | 5.463747<br>5   | 0.001563<br>806 | 5.576953<br>456 | 0.265346<br>417 |
|                   | 322.1 - 314.1   | 64.00416<br>577 | 1.07172<br>E-08 | 8.000260<br>355 | 0.000103<br>522 |

## VI. DISCUSSION

One of the important traits of a Multiloop controller is the smooth removal of interaction among the loops thereby providing the operator with the freedom to operate near the limits. The SISO control design conveniently ignores the interaction effect and thereby made the design easy at the cost of performance degradation. As the CSTR is operated as a MIMO system, the interaction of change in the temperature on the concentration and vice versa are dominant. The performance of the designed Multiloop PI controller using the SBL design method is analysed by intentionally perturbing the CSTR with setpoint and disturbances at three different operating conditions. The performance is quantified for the control of both the concentration and temperature with ISE and IAE. The effect of interaction is captured and presented alongside the servo and regulatory response plots at various operating conditions. They show that interaction plays a meagre role under SBL control and hence only a slight deviation from the nominal values are recorded. By a scrutiny of the response plots along with the performance metrics it is found that the suggested Multiloop control based on SBL design method performed indubitably well under various test conditions as presented in the paper.

## VII. CONCLUSION AND FUTURE WORK

A Multiloop control philosophy is tried on the CSTR utilizing a powerful SBL design technique. One interesting feature of the SBL method lies in its simple design procedure, which involves considering only the diagonal elements, hence the interaction dynamics are discarded during its design phase. As an extension of this work, Multivariable control strategies can be implemented to analyze the interaction effects.

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APPENDIX A

|       |                                      |
|-------|--------------------------------------|
| MIMO  | Multi Input Multi Output             |
| SISO  | Single Input Single Output           |
| CSTR  | Continuous Stirred Tank Reactor      |
| BLT   | Biggest Log modulus Tuning           |
| PI    | Proportional + Integral              |
| PID   | Proportional + Integral + Derivative |
| RGA   | Relative Gain Array                  |
| FOPDT | First Order Plus Dead Time           |
| SBL   | Stability Boundary Locus             |
| ISE   | Integral square error                |
| IAE   | Integral Absolute Error              |
| $K_C$ | Proportional Gain                    |
| $K_i$ | Integral Gain                        |