Estimation of Varying Reaction Times with RNN and Application to Human-like Autonomous Car-following Modeling

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***Abstract*—The interaction between human-driven vehicles and autonomous vehicles has become a vital issue in micro-transportation science. Compared to autonomous vehicles, human-driven vehicles have varying reaction times that could compromise traffic efficiency and stability. But human drivers can anticipate future traffic conditions subconsciously, which guarantees qualified performance. This paper proposes an estimation method of varying reaction times and a human-like autonomous car-following model. The varying reaction times are estimated based on recurrent neural networks (RNNs) after the cross-correlation analysis of human-driven vehicles' trajectory profiles. A human-like autonomous car-following model is established based on Intelligent Driver Model (IDM), considering both varying reaction times and temporal anticipation, and the short form is IDM_RTTA. The analytical string stability of IDM_RTTA is deduced and illustrated. The trajectory simulation result shows that increasing accuracy of trajectory prediction is obtained with the proposed model, which will benefit the interaction between human-driven vehicles and autonomous vehicles.**

***Keywords*—Car-following model; intelligent driver model; human-driven vehicle; autonomous vehicle; varying reaction time; string stability**

I. Introduction

Autonomous driving technology has developed rapidly, and with the gradually pervading use of autonomous vehicles, road traffic will experience the coexistence of human-driven and autonomous vehicles. In this situation, the interaction between human-driven vehicles and autonomous vehicles has become a vital issue in micro-transportation science. Due to the nature of human characteristics, the car-following behavior of human-driven vehicles differs from autonomous driving implemented in most microscopic models, which is a controversial topic in traffic flow analysis and simulation [1].

Reaction time is a widely recognized human driver factor, which has been incorporated into car-following modeling. Although these contributions of human-driven vehicles' reaction time have great achievement in the investigation of human characteristics and autonomous driving, the reaction time considered in car-following modeling is usually assumed as one or several constants without elaborately estimated. Furthermore, to the best of the authors' knowledge, few studies consider the inter-driver heterogeneity as well as the intra-driver heterogeneity (i.e., one driver's reaction times change from event to event [2]) of reaction times, in this case, the varying feature of reaction times is not deeply investigated and incorporated into car-following models.

To bridge these gaps, in this paper, we suggest a data-driven method to estimate the varying reaction time based on real human driving data. The estimation method includes first analyzing human-driven vehicles' trajectory profiles with cross-correlation and then learning and predicting reaction times with recurrent neural networks (RNN), which can reflect the inter-driver and intra-driver heterogeneities of reaction time. Moreover, the estimated varying reaction times are applied to car-following modeling by extending the intelligent driver model (IDM). The proposed model shows qualified analytical string stability and simulation accuracy.

The rest of this paper is organized as follows. Section II deploys the literature review of reaction time estimation and the corresponding car-following models. The estimation method for varying reaction times and the modified IDM is proposed in Section III. The estimation results are applied to the proposed car-following model, and the stability analysis and trajectory simulation are conducted, which is presented in Section IV. Section V gives the conclusion.

II. Literature Review

As car-following behavior is essential to microscopic traffic research, recent studies attempt to incorporate human characteristics into car-following models for the in-depth investigation of human-driven vehicles. Compared to autonomous vehicles, human-driven vehicles, on the one hand, have reaction times that could compromise traffic efficiency and stability. On the other hand, unlike machines, human drivers can anticipate future traffic conditions subconsciously, which guarantees qualified performance. Reaction time is composed of mental processing time, body movement time, and vehicle response time [3]. Multiple studies used real trajectory data for analysis to estimate the reaction time, and there was agreement that reaction time can be estimated by the gap between stimulus and response [4], [5], [6], [7], [8]. A hybrid model is proposed by [9], which estimates the desirable acceleration of the driver by car-following model then estimates the reaction time by an artificial neural network. Reference [10] calibrated the reaction time as an constant using the field data of the intersections.

As a widely recognized human driver factor, reaction time has been incorporated into car-following modeling. The popularly used safety distance car-following model, Gipps model, involved a constant reaction time [11]. Gazis-Herman-Rothery (GHR) model [12] incorporated reaction time and was extended by [13] considering the inter-driver heterogeneity of reaction time. Optimal velocity (OV) model that takes reaction time into account was presented in [14], and was modified by [15] with small reaction times and long reaction times considered separately for realistic performance. Reference [16] incorporated reaction time into car-following model and analyzed the impact of reaction time on traffic flow linear stability. Meanwhile, as the importance of human drivers' anticipation demonstrated by [17], the temporal anticipation was used as compensation to balance the negative effect of reaction time for stability [17], [18], [19], [20], [21].

III. Methodology

A. Human-Driven Vehicles' Reaction Time

Reaction time refers to the fact that humans have an inevitable time delay in decision-making and actions, such as driving a car. Most studies presented there is a time lag from vehicle speed profile to acceleration profile and inclined to use this time lag to define the reaction time. However, some have recognized that other information in the trajectory profile is also crucial to reaction time estimation, such as gap distance [6].

Time headway is an important indicator for evaluating driving safety, closely related to traffic flow composition and driving behavior. Time headway represents the time difference between two vehicles passing through the same place. It can be calculated by dividing the headway of two vehicles (i.e., from the leading vehicle's front bumper to the subject vehicle's front bumper) by the speed of the subject vehicle, presented in (1). As time headway includes the information of gap distance as well as velocity, it can be regarded as the stimulus and acceleration as the response. Therefore, we define reaction time (T) as the time lag between time headway and acceleration, as shown in Fig. 1.

$$thw_t = \frac{h_t}{v_t} \tag{1}$$

where thw_t and h_t are the time headway and headway between the subject vehicle and the leading vehicle at time t. v_t is the speed of the subject vehicle at time t.

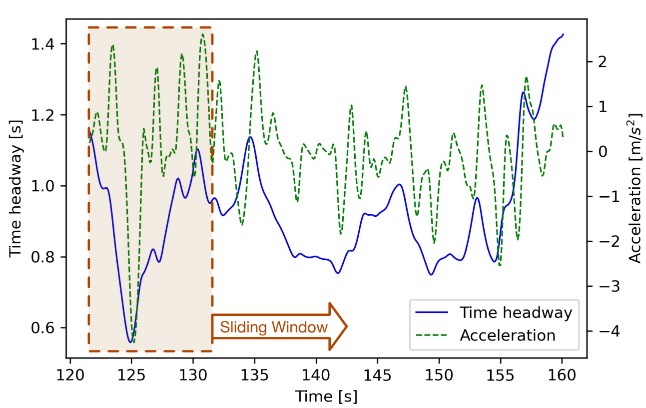


Fig. 1. The Profile of Time Headway and Acceleration.

B. Estimation of Varying Reaction Times based on RNN

Since it is difficult to obtain the time lags by measuring them manually, we apply the cross-correlation analysis as presented in (2)-(4) to estimate them [12]. As capturing the time-varying feature of reaction times, a sliding window is set to follow the time lags over time. To reasonably set the minima, maxima, and the length of the sliding window for estimation, we investigated the recent studies and summarized them in [22]. We assume the reaction time is distributed from 0.4 s to 3.0 s, i.e. $\alpha = 4$ and $\beta = 30$, and set the length of the sliding window as 10 s, which could capture the time-varying feature of reaction time, as illustrated in Fig. 1.

$$R_\tau = \mathrm{E}[thw_t a_{t+\tau}] \tag{2}$$

$$\rho_\tau = \frac{R_\tau - \mu_{thw}\mu_a}{\gamma_{\Delta v}\gamma_a} \tag{3}$$

$$\tau^* = \{\tau |\max(\rho_\tau),\ \alpha \le \tau \le \beta\} \tag{4}$$

where R_τ represents the cross-correlation function. $\mathrm{E}[\cdot]$ denotes the expectation function. thw_t and $a_{t+\tau}$ are the time headway and acceleration sequences. μ_{thw} and μ_a denote the mean values of the time headway and acceleration sequences respectively. γ_{thw} and γ_a are the standard deviations of the sequences. α and β are the minima and maxima of the reaction time, as analyzed and assumed above.

With the cross-correlation method, we are able to collect a dataset that contains the trajectory and the corresponding reaction time. We further apply a recurrent neural network (RNN), which is good at processing sequence learning problems, to learn the varying reaction times from the dataset. Fig. 2 shows the architecture of a typical RNN that takes on an input sequence to generate an output, and the inputs are in order. The RNN learns the hidden sequence order and the corresponding output value, as presented in (5)-(6). The input X_t is the trajectory data, including gap distance (Δx_t), relative speed (Δv_t), speed (v_t), acceleration (a_t), and time headway (thw_t). The output O_t is the reaction time (T). Thus, the varying reaction times can be estimated from a given trajectory.

$$S_t = tanh(U \cdot X_t + W \cdot S_{t-1}) \tag{5}$$

$$O_t = f(V \cdot S_t) \tag{6}$$

where U is the weight matrix from the input layer to the hidden layer. W is the weight matrix considering historical input data. V is the weight matrix from the hidden layer to the output layer.

C. Car-Following Model with Varying Reaction Times

The Intelligent Driver Model (IDM) was first proposed by [23] to simulate bottleneck congestion, as formulated in Eqs.(7)-(8). It is a distinguished mathematical car-following model that provides collision-free behavior as well as self-organizing properties, which can be used in adaptive cruise control (ACC) [24].

$$\dot{v}_t = \tilde{a}\left[1 - \left(\frac{v_t}{\tilde{v}}\right)^4 - \left(\frac{S(v_t, \Delta v_t)}{\Delta x_t}\right)^2\right] \quad (7)$$

$$S(v_t, \Delta v_t) = s_0 + t_0 v_t - \frac{v_t \Delta v_t}{2\sqrt{\tilde{a}\tilde{b}}} \quad (8)$$

where $S(v_t, \Delta v_t)$ denotes the desired space headway function that is obtained from the vehicle's speed (v_t) and relative speed (Δv_t). Δx_t is the gap distance between the subject vehicle and the leading vehicle at time t, which differs from h_t and can be calculated by $h_t - l$, in which l is the length of the leading vehicle. The desired speed ($\tilde{v}$), the maximum acceleration ($\tilde{a}$), the maximum deceleration ($\tilde{b}$), the desired time headway (t_0), and the minimum space headway (s_0) are the model parameters need to be calibrated.

We extend IDM based on varying reaction times and temporal anticipation to establish a human-like autonomous car-following model, designated as IDM_RTTA. Reference [17] suggested applying the constant-acceleration heuristics method to calculate future velocity. According to this concept, we formulated temporal anticipation in (9)-(11), which predict the future t timestep based on the trajectory at $t - T$ timestep. Therefore, for the estimation result of varying reaction time (T), the extended IDM function is established in (12), which can be further expanded as in (13).

$$\Delta x'_t = \Delta x_{t-T} + \Delta v_{t-T} T \quad (9)$$

$$\Delta v'_t = \Delta v_{t-T} - \dot{v}_{t-T} T \quad (10)$$

$$v'_t = v_{t-T} + \dot{v}_{t-T} T \quad (11)$$

$$\dot{v}_t = f(\Delta x', \Delta v', v')_t = f(\Delta x, \Delta v, v, \dot{v})_{t-T} \quad (12)$$

$$\dot{v}_t = \tilde{a}\left[1 - \left(\frac{v_{t-T} + a_{t-T}T}{\tilde{v}}\right)^4 - \left(\frac{s_0 + t_0(v_{t-T} + a_{t-T}T) - \frac{(v_{t-T}+a_{t-T}T)(\Delta v_{t-T} - a_{t-T}T)}{2\sqrt{\tilde{a}\tilde{b}}}}{\Delta x_{t-T} + \Delta v_{t-T}T}\right)^2\right] \quad (13)$$

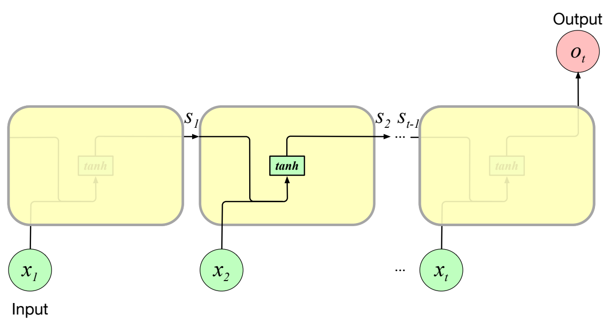


Fig. 2. RNN Architecture.

IV. EXPERIMENTS

A. Data Preparation

Analysis and simulation for human-driven vehicle interaction with autonomous vehicles require in-depth insights into human-driven vehicle behavior based on real-world human-driven vehicle trajectory data. The NGSIM dataset [25] is human-driven vehicle trajectory data collected from the real world, extracted from high-definition video by the Next Generation Simulation (NGSIM) computer program. The Interstate 80 (I-80) freeway dataset was collected on April 13, 2005, on eastbound I-80 in the San Francisco Bay Area in Emeryville, California, which was conducted under the NGSIM program. This study area has one high-occupancy lane (lane 1) and five regular lanes (from lane 2 to lane 6), including an on-ramp, and the length of this area is 503 m, as shown in Fig. 3. The author in [26] reconstructed this dataset to meet the consistency of vehicle kinematics and the reasonability of microscopic traffic dynamics. We extract car-following events from this reconstructed dataset to analyze the behavior of the human-driven vehicle and establish human-like autonomous driving. We obtain 1338 car-following events involving 636842 trajectory data points, and the trajectory resolution is 10Hz, i.e., the time interval between two adjacent time steps is 0.1 s. To ensure the testing process is independent, we select the car-following events in Lane 2 involving 330 vehicle pairs as testing data. Thus, 1008 vehicle pairs' car-following events in the rest of the lanes (from Lane 3 to Lane 6) are used as training data.

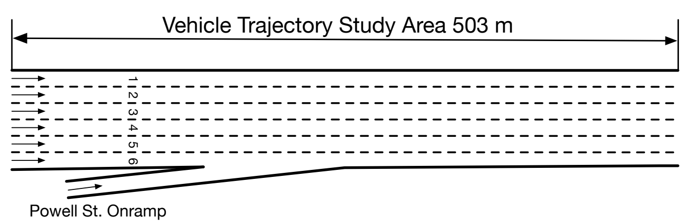


Fig. 3. Schematic of Eastbound/Northbound I-80.

B. Varying Reaction Times and Model Calibration

The time lags are obtained from the training dataset with the cross-correlation method and used as output data in the RNN training process. In this process, the RNN model can learn to output the reaction times from trajectory profiles, and its internal parameters update with the training samples for better output accuracy. The configuration of RNN is validated and summarized as follows:

TABLE I. Calibrated Parameters of IDM and IDM_RTTA

T	$\tilde{a}$	$\tilde{b}$	$\tilde{v}$	t_0	s_0
/	1.05	1.76	27.14	1.25	2.09
0.4	1.89	2.60	25.29	2.75	4.05
0.5	2.58	1.72	29.14	2.53	5.66
0.6	2.36	2.71	29.53	2.19	2.90
0.7	2.28	1.80	26.77	2.28	2.29
0.8	2.34	2.63	29.93	2.68	4.76
0.9	2.07	1.29	28.50	2.56	3.23
1.0	2.09	2.40	27.12	2.98	2.33
1.1	1.51	1.29	27.85	2.17	2.08

- Number of hidden layers: 1
- Number of neurons: 10
- The cost function: mean squared error
- The optimization algorithm: Adam [27]
- Batch size: 32
- Epochs: 20

The well-trained RNN model is applied to testing data. The estimated reaction times are obtained, and their percentages are illustrated in Fig. 4. The result indicates that this method could estimate the varying reaction times from trajectory profiles. The great majority of reaction times distribute from 0.4 s to 1.1 s. Then, this method will be integrated into the extended IDM model (IDM_RTTA) for online estimation to imitate human-driven vehicle behavior as human-like as possible.

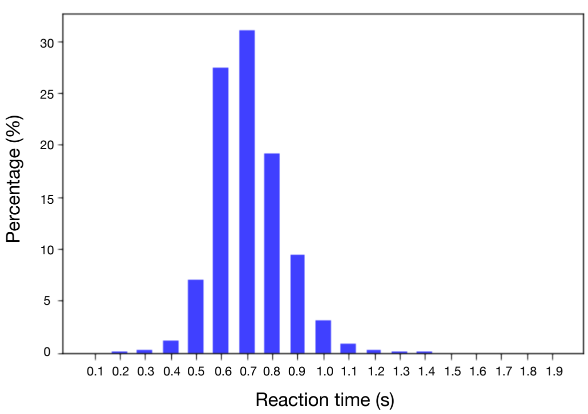
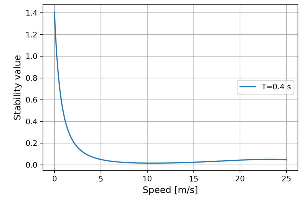


Fig. 4. Distribution of Dynamic Reaction Times.

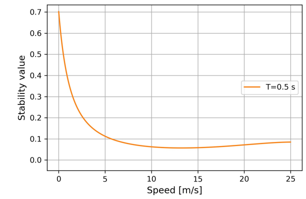
We calibrate the proposed IDM_RTTA for different reaction times ($T = 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1$) with the Genetic Algorithm (GA) [28], which is a stochastic global search optimization algorithm. The calibrated results are presented in Table I. The parameters of IDM are also calibrated, as shown in the first line of this table, for further model comparison.

C. Stability Analysis

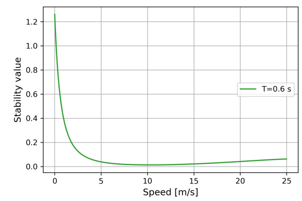
The stability of car-following models is a hot topic in the studies of traffic flow theory [29]. A car-following model can be simply represented by a control equation as in (12), and its equilibrium situation is presented in (14).



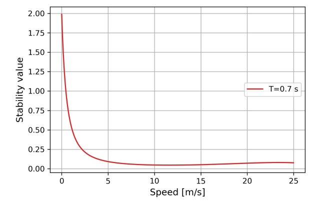
(a) Reaction Time is 0.4 s



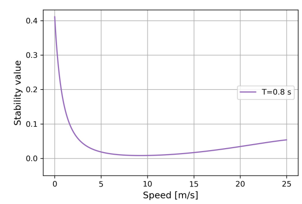
(b) Reaction Time is 0.5 s



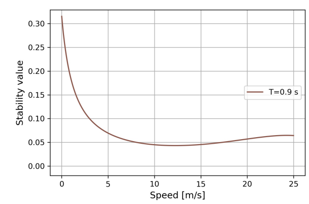
(c) Reaction Time is 0.6 s



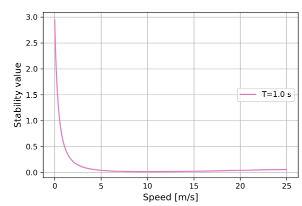
(d) Reaction Time is 0.7 s



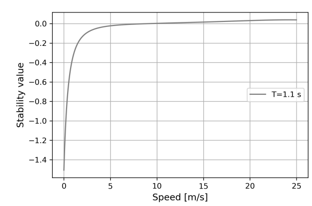
(e) Reaction Time is 0.8 s



(f) Reaction Time is 0.9 s



(g) Reaction Time is 1.0 s



(h) Reaction Time is 1.1 s

Fig. 5. String Stability for Varying Reaction Times.

$$f(\Delta x^*_{t-T}, 0, v^*_{t-T}, 0) = 0 \tag{14}$$

where acceleration and relative speed are zero in equilibrium situation, i.e., $\dot{v}_t = a_{t-T} = 0$ and $\Delta v_{t-T} = 0$, and there is an equilibrium solution for gap distance and speed, i.e., $\Delta x_{t-T} = \Delta x^*_{t-T}$ and $v_{t-T} = v^*_{t-T}$.

Empirical observations suggest that the unstable speed-spacing relationship can manifest in traffic flow as only one vehicle deviates from equilibrium (e.g., a slight change in speed) because the perturbations propagate to upstream traffic [23]. If a car-following model has string stability, for an infinite platoon of vehicles in equilibrium, the disturbance will decay as it propagates upstream [23], [30]. Although there are plenty of studies on the stability of the car-following models [1], as the IDM_RTTA model is built with varying reaction times, the impact of this new variable on the string stability of traffic flow needs to be explored.

The research in [30] supposed a small deviation from the equilibrium state and deduced the control equations. The unstable condition are given in (15):

$$\frac{1}{2}{f_v}^2 - f_{\Delta v} f_v - f_{\Delta x} < 0 \tag{15}$$

where $f_{\Delta x}, f_{\Delta v}, f_v$ are the partial differential of the coupled differential equation for gap distance, relative speed, and speed, which can be calculated by (16).

$$\begin{cases} f_{\Delta x} = \frac{\partial f(\Delta x, \Delta v, v, a)}{\partial \Delta x}\Big|_{(\Delta x^*_{t-T}, 0, v^*_{t-T}, 0)} \\ f_{\Delta v} = \frac{\partial f(\Delta x, \Delta v, v, a)}{\partial \Delta v}\Big|_{(\Delta x^*_{t-T}, 0, v^*_{t-T}, 0)} \\ f_v = \frac{\partial f(\Delta x, \Delta v, v, a)}{\partial v}\Big|_{(\Delta x^*_{t-T}, 0, v^*_{t-T}, 0)} \end{cases} \tag{16}$$

This string stability analysis method is applied to IDM_RTTA, and the following partial differential results are derived in (17). Combining (15) and (17), the string stability of IDM_RTTA can be evaluated, and for the varying reaction times and corresponding parameters presented in Table I, the stability value against equilibrium speed is plotted in Fig. 5. This clearly visualizes that the calibrated IDM_RTTA with varying reaction times stays stable at most circumstances. The model is unstable for equilibrium speeds below 10 m/s when reaction time is 1.1 s, as shown in Fig. 5h.

$$\begin{aligned} f_{\Delta x} &= 2\tilde{a}\frac{(s_0 + t_0 v^*_{t-T})^2}{{\Delta x^*_{t-T}}^3} \\ f_{\Delta v} &= \sqrt{\frac{\tilde{a}}{\tilde{b}}}\frac{v^*_{t-T}(s_0 + t_0 v^*_{t-T})}{{\Delta x^*_{t-T}}^2} - 2\tilde{a}\frac{T(s_0 + t_0 v^*_{t-T})^2}{{\Delta x^*_{t-T}}^3} \\ f_v &= -2\tilde{a}\left[\frac{2}{\tilde{v}}\left(\frac{v^*_{t-T}}{\tilde{v}}\right)^3 + \frac{t_0(s_0 + t_0 v^*_{t-T})}{{\Delta x^*_{t-T}}^2}\right] \end{aligned} \tag{17}$$

D. Trajectory Simulation

Trajectory simulation enables measuring the driving models' operation accuracy by comparing them to human-driven trajectories. The comparison is usually evaluated by a measure of performance (MoP), as shown in (18). The simulated trajectory (speed-time and space-time profiles) can be calculated by a discrete-time car-following process based on the acceleration obtained from IDM_RTTA, as formulated in (19). Moreover, the mean square error (MSE) of the simulated and observed trajectories is calculated.

$$\text{MSE} = \frac{1}{M}\sum_{t=1}^{M}[x_t - \hat{x}_t]^2 \tag{18}$$

$$\begin{cases} \hat{v}_t = \hat{v}_{t-1} + \hat{a}_t \Delta t \\ \hat{x}_t = \hat{x}_{t-1} + \hat{v}_{t-1}\Delta t + \frac{1}{2}\hat{a}_t \Delta t^2 \end{cases} \tag{19}$$

Where x_t denotes the observed location. $\hat{a}$ denotes the predicted acceleration. $\hat{v}$ and $\hat{x}$ denote the estimated speed and location.

TABLE II. MEASURE OF PERFORMANCE (MSE)

Model	Mean	SD
IDM	34.37	75.66
IDM_RTTA	29.75	57.95

Trajectories are simulated on the testing data (330 car-following events) with IDM and IDM_RTTA. The statistical metrics of MSE are compared (see Table II), and the trajectory profiles of one randomly selected car-following event are shown in Fig. 6. IDM is a mathematical car-following model with excellent performance, and simulations show that it can provide reliable results for trajectory reproduction. Their MSE values distribute with different mean values (34.37 for IDM and 29.75 for IDM_RTTA). Moreover, the standard deviation (SD) suggests that the IDM_RTTA model has a smaller range of MSE values and a denser distribution pattern. These results indicate that the extended human-like car-following model not only produces higher accuracy in reproducing trajectories but also shows a more stable driving quality.

V. CONCLUSION

In conclusion, we focus on estimating varying reaction times and the human-like autonomous car-following modeling. We investigate the existing reaction time estimation methods and identify the importance and possibility of incorporating human-driven vehicles' time-varying reaction times and temporal anticipation properties into autonomous car-following modeling. The extended IDM (IDM_RTTA) shows qualified string stability and demonstrates higher simulation accuracy for longitudinal control. Although the high-fidelity NGSIM data used in this paper is suitable for discovering human driver behavior, the proposed estimation method for varying reaction times still needs to be validated and tested on more real-world datasets, which is the direction of future work.

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REFERENCES

[1] M. Saifuzzaman and Z. Zheng, "Incorporating human-factors in car-following models: a review of recent developments and research needs," *Transportation research part C: emerging technologies*, vol. 48, pp. 379–403, 2014.

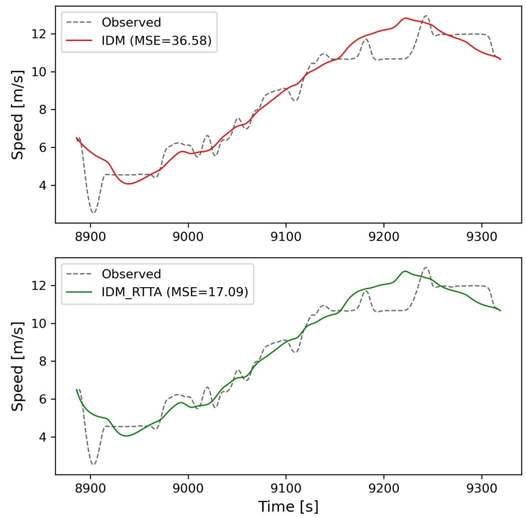
[2] T. Kim and H. Zhang, "A stochastic wave propagation model," *Transportation Research Part B: Methodological*, vol. 42, no. 7-8, pp. 619–634, 2008.

[3] M. Green, "" how long does it take to stop?" methodological analysis of driver perception-brake times," *Transportation human factors*, vol. 2, no. 3, pp. 195–216, 2000.

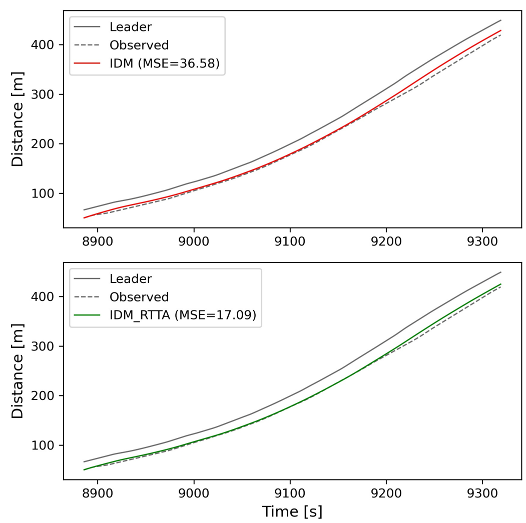
[4] R. E. Chandler, R. Herman, and E. W. Montroll, "Traffic dynamics: studies in car following," *Operations research*, vol. 6, no. 2, pp. 165–184, 1958.

[5] X. Ma and I. Andréasson, "Driver reaction delay estimation from real data and its application in gm-type model evaluation," *Transportation Research Record*, no. 1965, pp. 130–141, 2006.

[6] J. Zheng, K. Suzuki, and M. Fujita, "Car-following behavior with instantaneous driver–vehicle reaction delay: A neural-network-based methodology," *Transportation research part C: emerging technologies*, vol. 36, pp. 339–351, 2013.



(a) Speed-Time Profiles



(b) Space-Time Profiles

Fig. 6. Comparison of Simulated Trajectories of One Randomly Selected Car-Following Event

[7] A. Sharma, Z. Zheng, J. Kim, A. Bhaskar, and M. M. Haque, "Estimating and comparing response times in traditional and connected environments," *Transportation Research Record*, vol. 2673, no. 4, pp. 674–684, 2019.

[8] M. Zhu, X. Wang, and J. Hu, "Impact on car following behavior of a forward collision warning system with headway monitoring," *Transportation research part C: emerging technologies*, vol. 111, pp. 226–244, 2020.

[9] M. Rafati Fard, S. Rahmani, and A. Shariat Mohaymany, "Incorporating instantaneous reaction delay in car-following models: a hybrid approach," *Transportation research record*, vol. 2675, no. 10, pp. 1297–1311, 2021.

[10] A. M. Rahimi, E. Salehi, and A. Mazaheri, "Calibration of car following model based on two parameters related to reaction time and reaction time at stop (case study: Isfahan metropolitan)," *Journal of Transportation Research*, vol. 19, no. 3, pp. 195–210, 2022.

[11] P. G. Gipps, "A behavioural car-following model for computer simulation," *Transportation Research Part B: Methodological*, vol. 15, no. 2, pp. 105–111, 1981.

[12] D. C. Gazis, R. Herman, and R. W. Rothery, "Nonlinear follow-the-leader models of traffic flow," *Operations research*, vol. 9, no. 4, pp. 545–567, 1961.

[13] K. I. Ahmed, "Modeling drivers' acceleration and lane changing behavior," Ph.D. dissertation, Massachusetts Institute of Technology, 1999.

[14] M. Bando, K. Hasebe, K. Nakanishi, and A. Nakayama, "Analysis of optimal velocity model with explicit delay," *Physical Review E*, vol. 58, no. 5, p. 5429, 1998.

[15] L. Davis, "Modifications of the optimal velocity traffic model to include delay due to driver reaction time," *Physica A: Statistical Mechanics and its Applications*, vol. 319, pp. 557–567, 2003.

[16] Z. Yao, T. Xu, Y. Jiang, and R. Hu, "Linear stability analysis of heterogeneous traffic flow considering degradations of connected automated vehicles and reaction time," *Physica A: Statistical Mechanics and Its Applications*, vol. 561, p. 125218, 2021.

[17] M. Treiber, A. Kesting, and D. Helbing, "Delays, inaccuracies and anticipation in microscopic traffic models," *Physica A: Statistical Mechanics and its Applications*, vol. 360, no. 1, pp. 71–88, 2006.

[18] A. Kesting and M. Treiber, "Calibrating car-following models by using trajectory data: Methodological study," *Transportation Research Record*, vol. 2088, no. 1, pp. 148–156, 2008.

[19] D. Sun, D. Chen, M. Zhao, W. Liu, and L. Zheng, "Linear stability and nonlinear analyses of traffic waves for the general nonlinear car-following model with multi-time delays," *Physica A: Statistical Mechanics and its Applications*, vol. 501, pp. 293–307, 2018.

[20] A. Jafaripournimchahi, W. Hu, and L. Sun, "An asymmetric-anticipation car-following model in the era of autonomous-connected and human-driving vehicles," *Journal of advanced transportation*, vol. 2020, 2020.

[21] V. Kurtc, I. Anufriev, and D. Trufanov, "Multi-anticipative car-following model with explicit reaction-time delay," *Mathematical Models and Computer Simulations*, vol. 13, no. 6, pp. 1109–1115, 2021.

[22] L. Ma and S. Qu, "A sequence to sequence learning based car-following model for multi-step predictions considering reaction delay," *Transportation research part C: emerging technologies*, vol. 120, p. 102785, 2020.

[23] M. Treiber, A. Hennecke, and D. Helbing, "Congested traffic states in empirical observations and microscopic simulations," *Physical review E*, vol. 62, no. 2, p. 1805, 2000.

[24] A. Kesting, M. Treiber, M. Schönhof, and D. Helbing, "Adaptive cruise control design for active congestion avoidance," *Transportation Research Part C: Emerging Technologies*, vol. 16, no. 6, pp. 668–683, 2008.

[25] FHWA, "The Next Generation Simulation (NGSIM) [Online]," Available: http://www.ngsim.fhwa.dot.gov/, 2008.

[26] M. Montanino and V. Punzo, "Trajectory data reconstruction and simulation-based validation against macroscopic traffic patterns," *Transportation Research Part B: Methodological*, vol. 80, pp. 82–106, 2015.

[27] D. P. Kingma and J. Ba, "Adam: A method for stochastic optimization," *arXiv preprint arXiv:1412.6980*, 2014.

[28] M. J. Kochenderfer and T. A. Wheeler, *Algorithms for optimization*. Mit Press, 2019.

[29] A. Talebpour and H. S. Mahmassani, "Influence of connected and autonomous vehicles on traffic flow stability and throughput," *Transportation Research Part C: Emerging Technologies*, vol. 71, pp. 143–163, 2016.

[30] J. A. Ward, "Heterogeneity, lane-changing and instability in traffic: A mathematical approach," Ph.D. dissertation, University of Bristol Bristol, UK, 2009.