Ọdịgbo Metaheuristic Optimization Algorithm for Computation of Real-Parameters and Engineering Design Optimization

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***Abstract*—This paper proposes a new population-based global optimization algorithm, Ọdịgbo Metaheuristic Optimization Algorithm–ỌMOA, for solving complex bounded-constraint/single objective real-parameter problems found in most engineering and scientific applications. It's inspired by the human socio-cultural informal discipleship learning pattern inherent in the behavior of the Ndịgbo peoples; the subject – primary (Nwa-ahịa), in mercantile cycle grows to a secondary (Mazi) owing to the intuitive stratagem (dialect - Ịgba) embedded in an aged-long cultural model "Ịgba-ọsọ-ahịa" (meaning, strategic marketing skills, and practice). The model mimics the search routine for satisfying a customer's need in the market, built into exploration and exploitation applied in the mathematical model. About 30 complex classical unconstrained functions are tested, comparing results with that of five similar state-of-the-art algorithms. Also, 29 CEC-2017 single objective real constraint benchmark serious dimensional problems were simulated and compared against the winners of that competition. Validation includes statistical (t-test, p-value) comparison and for 50 Dimension constraint problems as ỌMOA demonstrated superior performance. TCS (9.18%), WBP (6.3%), PVDP (601%), RGP (319%), RBP (760%), GTCD (202%), HIMMELBLAU (4%), and CDP (88.12%) are the improvements made on 8 CEC-2020 engineering real design problems against the former best performances; OMOA is simple to implement, replicate and applicable across domains. Also, some new, improved optimum was obtained in Shubert and Schaffer 4 function compared to the global optimums.**

Keywords*—Human socio-cultural; nature-inspired; informal-learning; global optimization

I. Introduction

Humans and animals face challenges within their time and space of habitation, and they attempt to solve the challenges by making decisions and selecting and combining variables influencing the conditions. The challenges range from simple to difficult-complex ones, but the satisfaction derived from attaining the goal motivates effort for solution pursuant [1]. Engineering has availed very good solutions for small scaled problems using exact methods, but such fails when the problem becomes special and high dimension, become very costly and time consuming [2]. Meanwhile, the study of nature showed complex problems solved by meta-ideas and heuristics. The aesthetics that describes the meta-heuristics provide solutions that are near-optimal yet scalable with problem dimensions [3] despite the difficult procedural uncertainties [4]; the huge difficulty is associated with the mapping of routines called intelligence from rules or heuristics that describe events of nature which falls in a multidisciplinary field [5, 6]. Research in this direction has yielded several methodologies for solving engineering problems, yet more are anticipated [6]. This work aims to address some multidisciplinary domain concerns; a significant gap in balancing exploitation and exploration in populations of solution search impacts the state-of-art. Also, most recent works have scantily described the critical analogies of the metaphors that reflect the aesthetics of the target nature's source with the derived mathematical models, while the majority favors hybridization. Also, only a handful of the existing algorithms had human behavior metaphors, which this work proposes. Based on life science, a simple category of existing solutions could be into biological and non-biological (abiotic) hybrids, Bio-Abiotic hybrids, Bio-Bio hybrids, and Abiotic – Abiotic hybrids; however other literature may use alternative categorizations such as Swarm, Evolutionary, and Human intelligence. Genetic algorithms (GA) led the natural biological methods [7]. Particle Swarm Optimization (PSO) is inspired by flocks of birds and schools of fish [8] [9]. A few others due to space constraints are; Artificial Bee Colony from bee foraging [10]; Ant Colony [11]. In literature, numerous applications of the metaheuristics includes scheduling, loading, packaging, design, and control [12], image processing, amongst numerous others. The abiotic category is based on artificial physical experiences, such as Tabu Search, which made use of the creation of a tabu list [13]; Water Evaporation Optimization (WEO), mimicking the evaporation of water [14, 15]; JAYA mimicking the gravitation towards success [15]; Atomic Orbital Search (AOS) [16], etc. Some modified/hybrids are; Grey Wolf and PSO [17] gave (GWO-PSO), MOGSABAT [18] from the multiobjective gravitational search algorithm, and the echolocation ability of the bat algorithm [19]. Many other metaheuristic methods can be found [20, 21]. ỌMOA is a new strategy proposed by this work; the data is from the human population shown in Section II. The aesthetics are based on informal learning. The mathematical relations are developed in Section III and experiments, results and discussions are also presented in Section III, while Section IV is the conclusion.

The data of this work is gathered from the Ndịgbo people's mercantilism. This ideology is found in major Market setups across the World, where Ndịgbo are found in huge populations

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[22, 23]. They cooperate and maintain this characteristic ideology they call "ịgba"; meaning stratagem. [24, 25]. The aesthetics; every male is disciple/given-chance-to-hands-on/learn informally in commerce backed by some form of agreement [26]; a model known to them as "Ịgba-ọsọ-ahịa" which means strategic marketing skills acquisition and practice. Ahịa (market) is a solution space and holds all history of exploitations and explorations through Ịgba-ọsọ-ahịa model [27]. There exist huge risks and sacrifices, but the Ndigbo tolerates them [27].

A. ỌMOA Algorithm Description

In the Ahịa environment, the ultimate is to become a Mazi; The initial population is generated randomly as Ahịa-size. This is the "initialization mode"; The readiness, practicing, discipline, trading, cooperation, and the reluctance of the agents (known as ụmụ-ahịa in Igbo) is adjusted against the new environment each day; [from start-transduction mode to update-matching mode].

II. Modeling Data and Aesthetics

The work started with a collection of data from a local ahịa; the data is found at https://data.mendeley.com/datasets/wt3vt72mph/1. A few assumptions and facts extracted from data include but are not limited to the following parameters:

1) NORMS: (i) Every Mazi Own at least one shop. (ii) Every Nwa-ahịa is attached to a Mazi, a shop, and an ahịa. With an agreement, (iii) Death or risk are inevitable etc.

2) AXUMES: (i) Every Nwa-ahịa must satisfy a certain set percent of discipleship requirements to become a Mazi.

3) FACTS and Probable: (i) Certain Nwa-ahịa may succeed, fail, die, or get impeded. (ii) Certain Mazi may become greedy and unjust. (iii) Certain Ụmụ-ahịa had gotten second and third chances to make up, and many ahịa exist.

B. Sample Size of Selected Market

Data in Table I shows a snapshot of the collection, and the values represent the sub-total in each case. For example, the column representing "Japan"; "JAPANLINE"; "shops:35"; Parts: [12: "Nissan", 23: "Toyota Accessories", NULL:" x"].

C. The Model - Ahịa

The visualization of the setup of ahịa as a system (inputs, process, and outputs) schematically looks like Fig. 1 (left, right)

Fig. 1(a) shows Ahịa [n+9] described in Fig. 1(b) explicitly; the lines show the nonlinear relationships. The inner layers are shops and are associated with entities enlisted. The local Ahịa are networked across major cities in Nigeria (Ibadan, Lagos, Onitsha, etc.) which affiliates to extensions in Countries like Japan and Germany. The Ahịa primary agent (humans) are ụmụ-ahịa, and secondary are ndi-ọsọ-ahịa, Mazi, Bankers, customers (Regular and Non-Regular), suppliers, forwarding and clearing and etc. Meanwhile, the number of decision variables in sales, storage, borrowing etc., varies with constraints of environments like the cash flow, religion, and local/global politics etc.; taking shop 9 – GermanyLine Fig. 2; it comprises 1 –Mazi, 5 – Ụmụ-ahịa, 1 – Onye-ọsọ-ahịa, 12 – Regular Customers, 100 – Emergency Customers and trading on Benz-Spare Parts as shown.

TABLE I. Sampled Data From a Market and Shop Distributions

Object	JAPAN	TOYOTA	GERMANY	ABA	BAMENDA
Mazi	35	43	101	122	199
Ụmụ-ahịa	210	250	591	696	1162
Ndi-ọsọ-ahịa	35	43	101	122	199
Regular Customers	2204	2786	5039	19264	23616
Emergency Customers	2859	4646	10100	15595	20168
Shops					
Nissan and Toyota Parts	12	x	x	x	x
Toyota Accessories	23	x	x	x	x
Toyota-Spare-Parts	x	43	x	x	x
Benz-Spare-Parts	x	x	22	x	x
Audi Parts	x	x	31	x	x
Gold and Volkswagen	x	x	24	x	x
Benz Engine	x	x	11	x	x
Benz-Dashboard and Accessories	x	x	13	x	x
Cloth and Okrika	x	x	x	122	x
Tokunbo Fridges & Phones	x	x	x	x	199

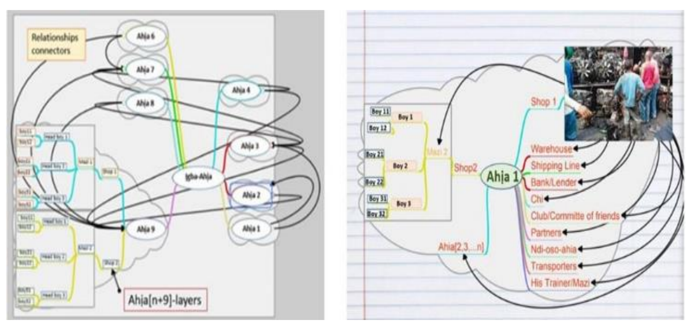
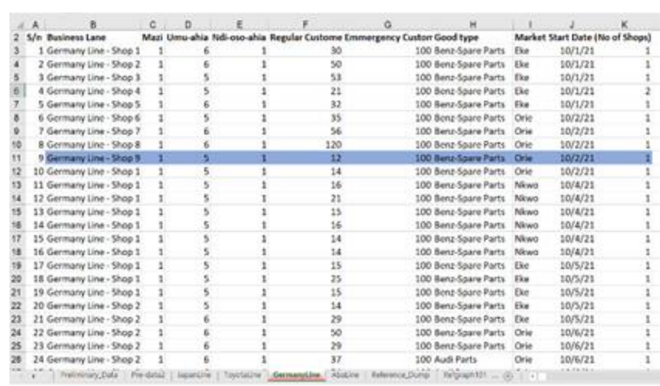


Fig. 1. Ahịa business model and networks.



S/n	Business Lane	Mazi	Umu-ahia	Ndi-oso-ahia	Regular Custome	Emmergency Custom	Good type	Market	Start Date	(No of Shops)
1	Germany Line - Shop 1	1	6	1	30	100	Benz-Spare Parts	Eke	10/1/21	1
2	Germany Line - Shop 2	1	6	1	50	100	Benz-Spare Parts	Eke	10/1/21	1
3	Germany Line - Shop 3	1	5	1	53	100	Benz-Spare Parts	Eke	10/1/21	1
4	Germany Line - Shop 4	1	5	1	21	100	Benz-Spare Parts	Eke	10/1/21	2
5	Germany Line - Shop 5	1	6	1	32	100	Benz-Spare Parts	Eke	10/1/21	1
6	Germany Line - Shop 6	1	5	1	35	100	Benz-Spare Parts	Orie	10/2/21	1
7	Germany Line - Shop 7	1	6	1	56	100	Benz-Spare Parts	Orie	10/2/21	1
8	Germany Line - Shop 8	1	6	1	120	100	Benz-Spare Parts	Orie	10/2/21	1
9	Germany Line - Shop 9	1	5	1	12	100	Benz-Spare Parts	Orie	10/2/21	1
10	Germany Line - Shop 1	1	5	1	14	100	Benz-Spare Parts	Orie	10/2/21	1
11	Germany Line - Shop 1	1	5	1	16	100	Benz-Spare Parts	Nkwo	10/4/21	1
12	Germany Line - Shop 1	1	5	1	21	100	Benz-Spare Parts	Nkwo	10/4/21	1
13	Germany Line - Shop 1	1	5	1	15	100	Benz-Spare Parts	Nkwo	10/4/21	1
14	Germany Line - Shop 1	1	5	1	16	100	Benz-Spare Parts	Nkwo	10/4/21	1
15	Germany Line - Shop 1	1	5	1	14	100	Benz-Spare Parts	Nkwo	10/4/21	1
16	Germany Line - Shop 1	1	5	1	14	100	Benz-Spare Parts	Nkwo	10/4/21	1
17	Germany Line - Shop 1	1	5	1	15	100	Benz-Spare Parts	Eke	10/5/21	1
18	Germany Line - Shop 1	1	5	1	25	100	Benz-Spare Parts	Eke	10/5/21	1
19	Germany Line - Shop 1	1	5	1	15	100	Benz-Spare Parts	Eke	10/5/21	1
20	Germany Line - Shop 2	1	5	1	14	100	Benz-Spare Parts	Eke	10/5/21	1
21	Germany Line - Shop 2	1	6	1	29	100	Benz-Spare Parts	Eke	10/5/21	1
22	Germany Line - Shop 2	1	6	1	50	100	Benz-Spare Parts	Orie	10/6/21	1
23	Germany Line - Shop 2	1	6	1	29	100	Benz-Spare Parts	Orie	10/6/21	1
24	Germany Line - Shop 2	1	6	1	37	100	Audi Parts	Orie	10/6/21	1

Fig. 2. Snap off idumọta sampling.

In obtaining the adjacency list from the data, assumptions made included (1) (1/0 means connected/not-connected) respectively; (2) also shop data is deterministic data at capture time, the network of the single shop nine(9) is modeled, and the simulation – Bayes graph is as shown

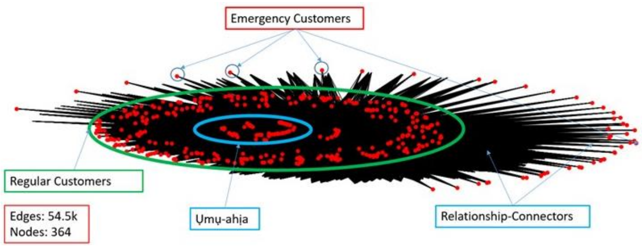


Fig. 3. The schematic representation of the network traffic.

The complex network of the shop (54500 edges, 364 nodes), on a market day named Ọrie; Fig. 3 is made. It depicts the intense cognitive field (energy) of nonlinear relationship maps responsible for transduced processes with experience (edges) of ụmụ-ahịa (nodes) that Ọrie day. Daily customer satisfaction time monotonically decreases with increasing edges, even with constraints in cycles. Beyond shop 9, thousands of shops contribute to the ahịa data; the computer structural model is shown in Fig. 4.

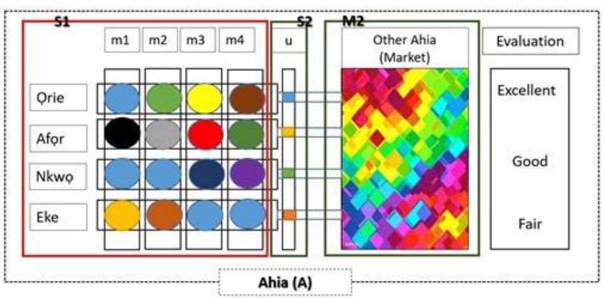


Fig. 4. Ahịa environment across many shops.

S1, S2, m1, m2, m3, m4, and M2 represents: shops 1, ndi-ọsọ-ahịa, nwa-ahịa-1, nwa-ahịa-2, nwa-ahịa-3, nwa-ahịa-4, and many other distance ahịa (markets). The updates are processes of the ụmụ-ahịa transforming on every market day (Ọrie, Afọr, Nkwọ, Eke). The colours is evidence uncertainty.

D. Initialization of Population (Market-Size)

ỌMOA; with the decision variables, Ịgba-ọsọ-ahịa and D-dimensions, the solution vector in the ahịa can be represented as (1).

$$Mazi = [x_1, x_2, x_3, ..., x_D] \tag{1}$$

The fitness value of each Mazi will be computed as a vector of (2);

$$f(Mazi) = f(x_1, x_2, x_3, .., x_D) \tag{1}$$

For instance, a new shop with new ụmụ-ahịa (ages of 3 and 8 yrs.), some constraints of this age group include (a) Nostalgic energy in early months, inherent childishness (comprises of untargetted and undirected energies): chaotic sleeping patterns, food pattern, and desires for the first few months persist. But discipleship (hands-on, disciplinary actions, corrections, task handling, rewards) between 1 and 5 years changes their energies to focused excitement; next is integrity and trust test; Each nwa-ahịa has a position and cost affected by such constraints and uncertainty in capacity, inductiveness, and reluctance. The population is generated using (3).

$$pop = \Phi(M, D) \tag{3}$$

The Φ is a random generator, M is the market population (pop), and D is the dimension. The pseudocode is shown below.

```
Initial parameters
Initialize the structure for the empty individuals
Initialize population array
while (not termination) Do
      generate uniform random population with
                    bounded size of market
      evaluate the cost of the individual
update individual population
end while
return best solution
```

E. Mathematics of Ịgba-ọsọ-ahịa

Fig. 5(a) in 3-dimensional space during the search is shown in 2-Dimension as Fig. 5 (b) as agents move to satisfy customers' scarce demand for "gold" and "leather" as in Fig. 4 to exploit S_1 and explore S_2.

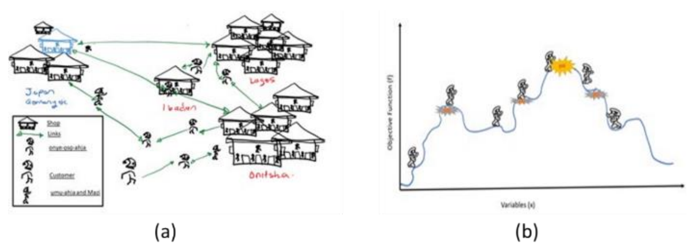


Fig. 5. Ịgba-ọsọ-ahịa cooperation by to find gold.

They cooperate, meet set thresholds, and satisfy the customer to get his Gold. Initializing a new Mazi, a new shop, and his contribution to the solution space will be given by (4).

$$U_n = f(X_1) \tag{2}$$

Combining equation (1) and (2);

$$U_n \leftarrow \begin{cases} x_i \in \{x_r\} \; mazi \; with \; new \; umu-ahia \\ x_i \in X_1 \quad 1 mazi, without \;\; umu-ahia \end{cases} \tag{3}$$

Some of the major constraints (3) as mentioned in Section II. n; the number of generations – the stopping criteria, r indicates x is random. Mazi cost alone in trade without ụmụ-ahịa in the cycle of Ahịa days (6) resolves to a fitness vector:

$$U_n = X_{n-1} * (P(i, :)) \tag{6}$$

Where $P(i,:)$ is the cost of Mazi in the population of P, i cycles of Ahịa days, and the transmute of Mazi's energies via the training processes. At the same time, the ụmụ-ahịa adopt the emitted energies originating from multidiscipline like phycology, social tactics, resilience, experience, transactional techniques, relationship with customers, banks, etc., The differences compared to theirs cut across domains and, by analogy, involve transduction [28]. This process is given by a resemblance of balancing potentials and kinetics.

$$T = rand(1, D) * (P(p,:)) - \tfrac{1}{2} * Ef * (X_r) \quad (4)$$

Where $P(p,:)$ in (4) is the cost of the new population at time p, Ef is the energy factor, while the X_r random ụmụ-ahịa cognitive state of five analogous Bayesian energies interacting actively in a shop.

$$X_r = X_{r+i} + 1/8 * Ef * (X_{r+i-1}) \quad (5)$$

$$X_{r+i-1} = X_{r+\Delta(i-1)} + 1/40 * Ef * X_{r+\Delta(i-1)} \quad (6)$$

$$X_{r+\Delta(i-1)} = 1/80 * X_{r+\Delta(i-n)} \quad (7)$$

Equations (5), (6) and (7) are all nonlinear cognitive vectors, and ratios of series [1/8, 1/40, and 1/80] of time divisions (could take any ratio as they are probabilities of random events, recall a state ranges from 0 to 1), i, r remain the same; visually, a huge network ensues as shown below.

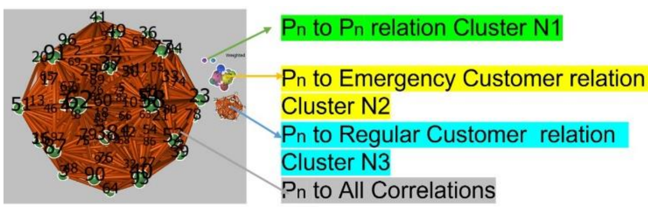


Fig. 6. Cognitive correlations between nwa-ahịa (PN) to any.

Fig. 6 clusters N1 ụmụ-ahịa with each other, and N2 and N3 are the clusters with customers. The probabilistic behavior of the interaction (edges) shows the very interesting transition of the ụmụ-ahịa (nodes) progress in the network in Fig. 7.

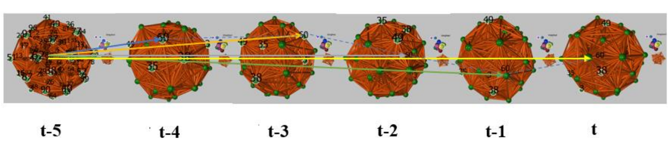


Fig. 7. Progress of cognitive signature on ụmụ-ahịa character.

Current time t; the previous timestamps as the selected node (60). Search in a generation gives (11).

$$X_r = X_{r+i} + 1/8 * Ef * (X_{r+\Delta(i-1)} + \cdots \; 1/40 * Ef(1/80 * X_{r+\Delta(i-n)}))) \quad (8)$$

Where updates at $r+i$ taken during iteration. The compact dynamics (12); mimics a rhythmic nodding to music and stratagem - ịgba, which gives.

$$U_n = U_{n-1} + T_{n-1} * X_r \quad (12)$$

Where U_n is a vector of emergent solutions. The threshold facilitates ụmụ-ahịa exploration; disciple-Rhythm - rD known as discipleship compliance, given by (13).

$$U(ij) = \begin{cases} u_m{}^j & \text{if } m \le rD \text{ OR } j = \delta \\ x_m{}^j & \text{if } m > rD \text{ AND } j \ne \delta \end{cases} \quad (9)$$

Where m is a random number [0,1], delta δ has the same dimension and size as the solution but is pseudorandom. This cooperation serves as the bond linking one source to another [23, 29]. Mazi; sometimes sacrifices profit for an improved - customer base and to escape the local optima trap by analogy as Ịgba-ọsọ-ahịa updates; objectives of the fitness bound the strategy as given in Fig. 8.

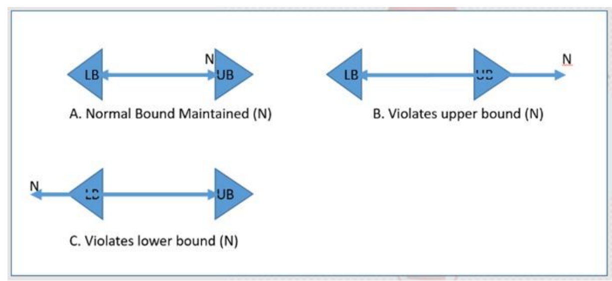


Fig. 8. Boundary strategy.

Constraints are bounded as in A, N collapse to upper-bound (UB) in B, and N collapse to lower-bound (LB) as shown in Fig. 8. Finally, methodic update results to best solutions shown.

$$\left.\begin{matrix} X_i = U_i \\ f_i = f_{U_i} \end{matrix}\right\} \text{if } f_{U_i} < f_i \quad (10)$$

$$X \text{ and } f \text{ remains the same if } f_{U_i} > f_i$$

Where (10); f_{Ui} is the fitness function from the best cost of the discipleship and adjustments made (error correction), while f_i is the best solution fitness of the original objective function, which is optimal.

F. Graphical Flow of ỌMOA

The ụmụ-ahịa can be considered as moving particles [30-32]. Mazi realization comes after generations of successful cycles [33]; rather than unhealthy competition, all ụmụ-ahịa depends on each other; The main body's pseudocode (2) during iteration is as follows:

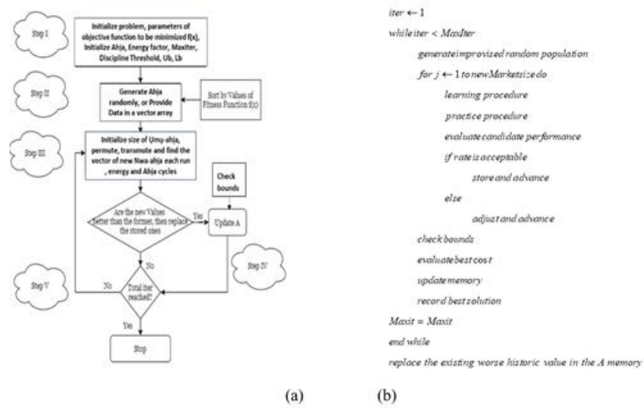


Fig. 9. Ọdịgbo metaheuristic optimization algorithm - ỌMOA and pseudocode.

The flow chart of Fig. 9 shows the methodology for applying the ỌMOA algorithm. The subsequent sections discuss applications.

III. Application of ỌMOA on Benchmark Functions

Most metaheuristic algorithms use Pattern Matrix, and the solutions are identified as those that improved through the number of generations up until the convergence time of the simulation. ỌMOA inherent energy synergy principles.

1) Default parameters are used

2) 30 independent runs were used for Unconstraint Benchmark, 50 for constraint functions. The parameters for the engineering designs are as stated in the referenced literature provided

3) The total number of cost function evaluations is 1000·n·M, where M = 10 is the number of iterations. The logarithmic Scale was considered for visualization due to its convenience and compact.

4) For the Constraint problems as depicted by the competition, a solution value less than 10^{-8} is treated as zero; several performance indicators for solution values are used: best, worst, mean, and standard deviation (Std). Test for convergence time also provided;

A. Experiments and Comparison of Results

ỌMOA is compared with five of the best similar algorithms as shown in Table II. Their codes are in the open domain/available online. The choice of only five is being mindful also of the limited space constraints to publish results.

TABLE II. Algorithms used for Validation

S/N	Algorithm	Ref	Category
1	Harris Hawks Optimization (HHO)-- 2019	[34]	Novel Idea
2	Moth Search (MS)-- -2018	[35]	Novel Idea
3	Elephant Herd Optimization (EHO)-- 2015	[36]	Novel Idea
4	LSHADE-SPACMA (A2) - 2017	[37]	Hybrid/Modified
5	EBOwithCMAR (A3) ---2017	[38]	Hybrid/Modified

The list in Table II is a competitive group; notably, 4 – 5 won the CEC 2017 competition [39, 40].

B. Experiment 1: Difficult Unconstraint Benchmark Functions

ỌMOA is validated on the existing established algorithms listed in Table II with about 30 difficult functions chosen with modality (unimodal to check and confirm exploitation strength, multimodal for diversity or exploratory capability of ỌMOA), Separability (possible separable and non-separable) and then multi-dimensionality (confirming search and exploratory strength of ỌMOA). The performance averages are visualized using boxplots. Further, the significance and statistical students test (t-test) was conducted for all algorithms, with a time complexity test. A subset of test benchmark functions with varying degrees of difficulty is used to substantiate that ỌMOA can exploit and explore the solution space and find the solutions for optimum. In Table III, unconstraint benchmark test functions are categorized in modality, Separability, and Dimensionality ***(N)***, also: ***M*** is the modality, ***0*** – Uni-modal; ***1*** – Multimodal, ***S*** is the Separability, ***0*** – Non-Separable; ***1*** – Separable.

TABLE III. Functions, Global Optimal Values, Bounds, and Dimensions

Fun(fn)	***Fun-name***	***SD***	***F(x*)***	***N***	***M***	***S***
F1	Step	$[-5.12, 5.12]^n$	0	30	0	1
F2	Sphere	$[-1, 1]^n$	0	30	0	1
F3	Sum Square	$[-5.12, 5.12]^n$	0	30	0	1
F4	Quartic	$[-6.0, 6.0]^n$	0	30	0	1
F5	Beale	$[-5.0, 5.0]^n$	0	2	1	0
F6	Easom	$[-100.0, 100.0]^n$	-1	2	1	0
F7	Matyas	$[-10.0, 10.0]^n$	0	2	1	0
F8	Colville	$[-10.0, 10.0]^n$	0	4	1	0
F9	Zakharov	$[-5.0, 5.0]^n$	0	30	1	0
F10	Schwefel 2.2	$[-10.0, 10.0]^n$	0	30	1	0
F11	Schwefel 1.2	$[-10.0, 10.0]^n$	0	30	1	0
F12	Dixon Price	$[-10.0, 10.0]^n$	0	30	1	0
F13	Bohachevsky 1	$[-100.0, 100.0]^n$	0	2	1	1
F14	Booth	$[-10, 10]^n$	0	2	1	1
F15	Holder Table	$[-10\ 10]^n$	-19.2085	2	1	1
F16	Michalewicz 2	$[0.0, \pi]^n$	-1.8013	2	1	1
F17	Michalewicz 5	$[0.0, \pi]^n$	-4.6877	5	1	1
F18	Michalewicz 10	$[0.0, \pi]^n$	-9.6602	10	1	1
F19	Rastrigin	$[-5.12, 5.12]^n$	0	5	1	1
F20	Schaffer2	$[-100, 100]^n$	0	2	1	0
F21	Schaffer 4	$[-100.0, 100.0]^n$	0	4	1	0
F22	Schaffer 6	$[-100.0, 100.0]^n$	0	6	1	0
F23	SixHumpCamelBack	[-5,5]	-1.0316	2	1	1
F24	Bohachevsky 2	$[-100.0, 100.0]^n$	0	2	1	0
F25	Bohachevsky 3	$[-100.0, 100.0]^n$	0	3	1	1
F26	Shubert	$[-10.0, 10.0]^n$	-186.73	5	1	1
F27	Drop Wave	$[-5.12, 5.12]^n$	-1	2	0	1
F28	Rosenbrock	$[-6.0, 6.0]^n$	0	2	0	0
F29	Griewank	$[-600.0, 600.0]^n$	0	30	1	0
F30	Ackley	$[-32.0, 32.0]^n$	0	2	1	0

C. Benchmark – Unimodal and Separable Functions

To tighten the competitiveness, we identified the algorithms with the highest performances with a t-value above 0.05. ỌMOA and A3 (all best solutions in BOLD font) lead with equal best performance as shown in Table IV. The MS was next, followed by A1, HHO, and WFS.

ỌMOA and A3 leading showed good exploration, and exploitation strength, particularly of ỌMOA obtained the best optimal objective solutions before the completion of generations. Fig. 10 also shows consistent distributions with fewer outliers.

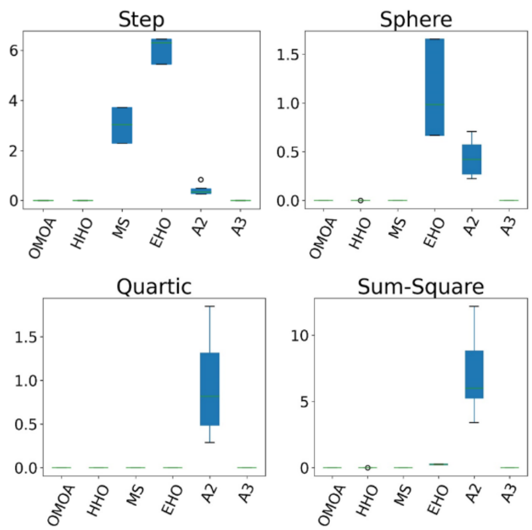


Fig. 10. Boxplot for unimodal and separable functions.

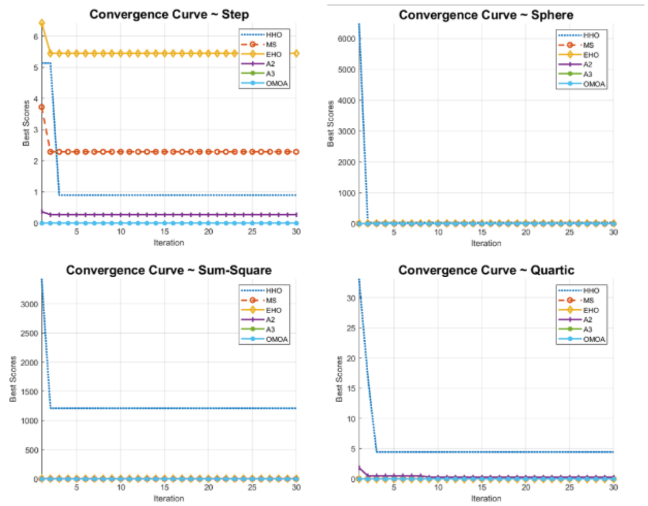


Fig. 11. Comparison of convergence curve for unimodal and separable functions.

Fig. 11 convergence comparison shows that ỌMOA, A3, A2, and WFS have faster and best convergences to the optimum in these problems while HHO lagged behind most of the time.

D. Unimodal and Non-Separable

The functions in this category include 30-dimensional problems Zakharov to Dixon Price with great complexity. Table V shows ỌMOA and A3 tops, followed by A2, A1, HHO, MS, and WFS. Besides Beale, which did not yield a better result, ỌMOA got even Easom, a problem with inherent complex nature.

TABLE IV. UNIMODAL AND SEPARABLE FUNCTIONS RESULTS

F(n)	Measure	ỌMOA	HHO	MS	EHO	A2	A3
F1	Best	**0.0E+00**	1.3E-06	2.3E+00	5.5E+00	1.5E-01	**0.0E+00**
n = 30	Worst	**0.0E+00**	7.6E-05	3.7E+00	6.4E+00	8.5E-01	**0.0E+00**
f = 0	Mean	**0.0E+00**	2.2E-05	3.0E+00	6.1E+00	4.1E-01	**0.0E+00**
	Sd	**0.0E+00**	3.0E-05	3.1E+00	6.1E+00	4.3E-01	**0.0E+00**
F2	Best	**0.0E+00**	0.0E+00	**0.0E+00**	6.7E-01	1.6E-01	**0.0E+00**
n = 30	Worst	**0.0E+00**	1.1E-07	**0.0E+00**	1.7E+00	7.1E-01	**0.0E+00**
f = 0	Mean	**0.0E+00**	1.1E-08	**0.0E+00**	1.1E+00	4.2E-01	**0.0E+00**
	Sd	**0.0E+00**	2.7E-08	**0.0E+00**	1.2E+00	4.1E-01	**0.0E+00**
F3	Best	**0.0E+00**	**0.0E+00**	0.0E+00	2.2E-01	2.5 E+00	**0.0E+00**
n = 30	Worst	**0.0E+00**	**1.2E-07**	7.2E-08	2.9E-01	1.2E+01	**0.0E+00**
f = 0	Mean	**0.0E+00**	**6.4E-09**	4.3E-08	2.5E-01	6.3E+00	**0.0E+00**
	Sd	**0.0E+00**	**2.4E-08**	5.3E-08	2.5E-01	6.7E+00	**0.0E+00**
F4	Best	**0.0E+00**	**0.0E+00**	**0.0E+00**	1.2E-06	2.1E-01	**0.0E+00**
n = 30	Worst	**0.0E+00**	**0.0E+00**	**0.0E+00**	2.2E-06	5.0E+00	**0.0E+00**
f = 0	Mean	**0.0E+00**	**0.0E+00**	**0.0E+00**	1.6E-06	1.3E+00	**0.0E+00**
	Sd	**0.0E+00**	**0.0E+00**	**0.0E+00**	1.6E-06	1.5E+00	**0.0E+00**

TABLE V. UNIMODAL AND NON-SEPARABLE RESULTS FOR TESTED ALGORITHMS

Function	Measure	QMOA	HHO	MS	EHO	A2	A3
F5	Best	5.7E-07	**0.0E+00**	6.0E-05	1.1E-05	0.0E+00	**0.0E+00**
n = 2	Worst	1.7E-01	**0.0E+00**	7.2E-04	2.3E-02	1.5E-05	**0.0E+00**
f = 0	Mean	4.7E-02	**0.0E+00**	2.9E-04	7.9E-03	1.0E-06	**0.0E+00**
	Sd	4.8E-02	**0.0E+00**	4.2E-04	1.3E-02	2.9E-06	**0.0E+00**
F6	Best	**-9.6E-01**	0.0E+00	7.7E-04	1.2E-01	0.0E+00	0.0E+00
n = 2	Worst	**-1.7E-235**	1.9E-05	1.3E-03	5.7E-01	0.0E+00	0.0E+00
f = -1	Mean	**-3.1E-01**	1.1E-06	1.0E-03	3.8E-01	0.0E+00	0.0E+00
	Sd	**3.6E-01**	3.6E-06	1.1E-03	4.2E-01	0.0E+00	0.0E+00
F7	Best	**0.0E+00**	**0.0E+00**	**0.0E+00**	4.3E-07	**0.0E+00**	**0.0E+00**
n = 2	Worst	**0.0E+00**	**0.0E+00**	**0.0E+00**	3.0E-06	**0.0E+00**	**0.0E+00**
f = 0	Mean	**0.0E+00**	**0.0E+00**	**0.0E+00**	1.9E-06	**0.0E+00**	**0.0E+00**
	Sd	**0.0E+00**	**0.0E+00**	**0.0E+00**	2.2E-06	**0.0E+00**	**0.0E+00**
F8	Best	**0.0E+00**	1.6E-06	1.8E-01	1.1E+00	**0.0E+00**	**0.0E+00**
n = 4	Worst	**0.0E+00**	7.7E-01	2.9E-01	3.2E+00	**0.0E+00**	**0.0E+00**
f = 0	Mean	**0.0E+00**	3.4E-02	2.4E-01	1.9E+00	**0.0E+00**	**0.0E+00**
	Sd	**0.0E+00**	1.4E-01	2.4E-01	2.1E+00	**0.0E+00**	**0.0E+00**
F9	Best	**0.0E+00**	0.0E+00	**0.0E+00**	3.6E-02	**0.0E+00**	**0.0E+00**
n = 30	Worst	**0.0E+00**	7.7E-07	**1.3E-08**	9.1E-02	5.1E-08	**0.0E+00**
f = 0	Mean	**0.0E+00**	8.0E-08	**4.3E-09**	6.0E-02	1.4E-08	**0.0E+00**
	Sd	**0.0E+00**	1.8E-07	**7.4E-09**	6.4E-02	2.1E-08	**0.0E+00**
F10	Best	**0.0E+00**	4.3E-07	**3.4E-05**	4.8E-01	6.4E-03	8.7E-06
n = 30	Worst	**0.0E+00**	2.7E-04	4.6E-04	6.2E-01	2.7E-02	4.3E-05
f = 0	Mean	**0.0E+00**	4.7E-05	1.9E-04	5.4E-01	1.8E-02	1.8E-05
	Sd	**0.0E+00**	7.6E-05	2.7E-04	5.5E-01	1.9E-02	2.1E-05
F11	Best	**0.0E+00**	0.0E+00	8.0E-08	3.4E+00	8.2E-02	**0.0E+00**
n = 30	Worst	**0.0E+00**	1.9E-03	3.8E-07	4.2E+00	5.5E-01	**0.0E+00**
f = 0	Mean	**0.0E+00**	9.4E-05	2.7E-07	3.8E+00	3.0E-01	**0.0E+00**
	Sd	**0.0E+00**	3.6E-04	3.0E-07	3.8E+00	3.4E-01	**0.0E+00**
F12	Best	**3.1E-09**	8.6E-02	6.7E-01	1.1E+00	6.7E-01	6.7E-01
n = 30	Worst	**6.7E-04**	2.6E-01	7.0E-01	1.3E+00	6.7E-01	6.7E-01
f = 0	Mean	**3.0E-05**	2.4E-01	6.9E-01	1.2E+00	6.7E-01	6.7E-01
	Sd	**1.2E-04**	2.4E-01	6.9E-01	1.2E+00	6.7E-01	6.7E-01

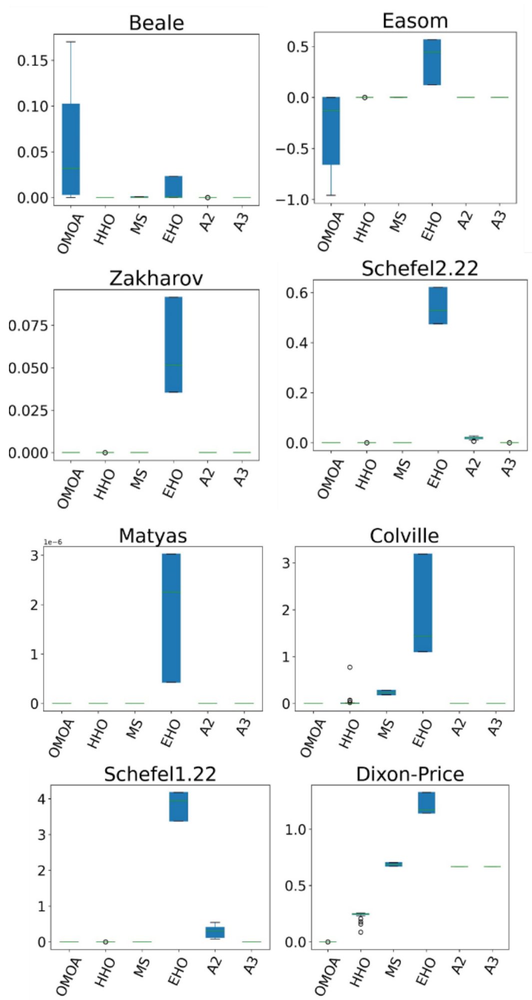


Fig. 12. Boxplot of unimodal and non-separable results.

The boxplot of Fig. 12 shows that the mean solutions distribution of the data of ỌMOA and A3 are tight, with little outliers equalling minimal deviation.

The convergence comparison of Fig. 13 confirms the summary made at the beginning of the subsection.

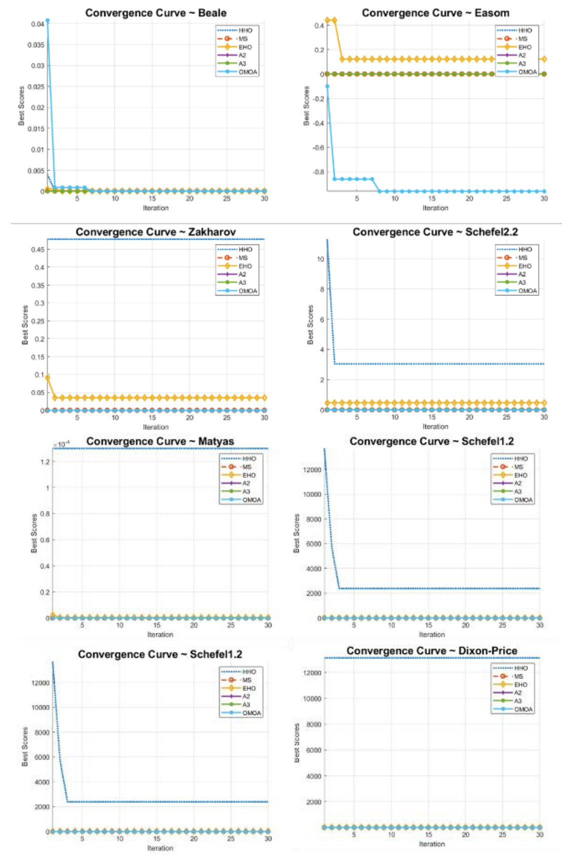


Fig. 13. Comparison of convergence curves for unimodal and non-separable function.

E. Multimodal and Separable

Complex structures, multiple, unequal hilltops, and valleys-shaped functions are tested as shown in Table VI. Besides the booth function, ỌMOA had remarkable exploratory abilities for the dimensionalities above n = 2 (i.e., n = 5, 10, 30) of the last three functions while tracking deeper than values provided by the global optima in literature for Holder Table, Michalewicz (2, 5, and 10). Rastrigin (n = 30) was also explored optimally by ỌMOA and HHO.

TABLE VI. MULTIMODAL AND SEPARABLE RESULTS

Function	Measure	ỌMOA	HHO	MS	EHO	A2	A3
F13	Best	**0.0E+00**	**0.0E+00**	2.8E-08	5.4E-03	**0.0E+00**	**0.0E+00**
n = 2	Worst	**0.0E+00**	**2.7E-08**	2.5E-07	1.5E-02	**0.0E+00**	**0.0E+00**
f = 0	Mean	**0.0E+00**	**3.3E-09**	1.7E-07	1.0E-02	**0.0E+00**	**0.0E+00**
	Sd	**0.0E+00**	**8.5E-09**	2.0E-07	1.1E-02	**0.0E+00**	**0.0E+00**
F14	Best	1.1E-04	0.0E+00	8.7E-06	3.8E-05	**0.0E+00**	**0.0E+00**
n = 2	Worst	2.6E-02	4.1E-04	4.1E-04	3.7E-02	**0.0E+00**	**0.0E+00**
f = 0	Mean	5.5E-03	9.1E-05	2.4E-04	2.0E-02	**0.0E+00**	**0.0E+00**
	Sd	5.9E-03	1.5E-04	3.0E-04	2.6E-02	**0.0E+00**	**0.0E+00**
F15	Best	**-5.0E+04**	0.0E+00	**-1.8E+01**	**-1.7E+01**	5.3E-06	0.0E+00
n = 2	Worst	**-2.5E+04**	1.3E-05	**-1.1E+01**	**-1.1E+01**	3.4E-04	0.0E+00
f = -19.2085	Mean	**-3.9E+04**	1.4E-06	**-1.5E+01**	**-1.4E+01**	1.0E-04	0.0E+00

	Sd	**1.2E+04**	3.4E-06	**2.8E+00**	**2.7E+00**	1.1E-04	0.0E+00
F16	Best	**-1.8E+00**	0.0E+00	1.2E-03	6.5E-03	0.0E+00	0.0E+00
n = 2	Worst	**-1.7E+00**	2.3E-06	7.4E-03	3.0E-02	0.0E+00	0.0E+00
f = -1.8013	Mean	**-1.8E+00**	2.9E-07	3.3E-03	1.7E-02	0.0E+00	0.0E+00
	Sd	**1.4E-02**	5.5E-07	4.4E-03	2.0E-02	0.0E+00	0.0E+00
F17	Best	**-4.0E+00**	1.5E-02	2.4E-01	9.0E-01	2.6E-06	0.0E+00
n = 5	Worst	**-2.8E+00**	1.2E+00	1.3E+00	1.2E+00	1.4E-04	4.4E-02
f = -4.6877	Mean	**-3.6E+00**	4.7E-01	8.5E-01	1.1E+00	4.0E-05	8.7E-03
	Sd	**2.8E-01**	6.1E-01	9.6E-01	1.1E+00	6.1E-05	1.8E-02
F18	Best	**-5.5E+00**	1.6E+00	3.4E+00	3.4E+00	8.6E-01	6.5E-01
n = 10	Worst	**-4.2E+00**	3.8E+00	4.9E+00	4.2E+00	1.3E+00	1.4E+00
f = -9.6602	Mean	**-4.7E+00**	2.9E+00	4.2E+00	3.9E+00	1.1E+00	1.1E+00
	Sd	**3.5E-01**	2.9E+00	4.2E+00	4.0E+00	1.1E+00	1.1E+00
F19	Best	**0.0E+00**	0.0E+00	0.0E+00	8.3E-01	9.2E+01	1.5E+01
n = 30	Worst	**0.0E+00**	3.3E-07	2.2E-07	1.2E+00	1.1E+02	7.3E+01
f = 0	Mean	**0.0E+00**	2.1E-08	1.2E-07	1.0E+00	1.0E+02	3.5E+01
	Sd	**0.0E+00**	6.5E-08	1.5E-07	1.0E+00	1.0E+02	3.9E+01

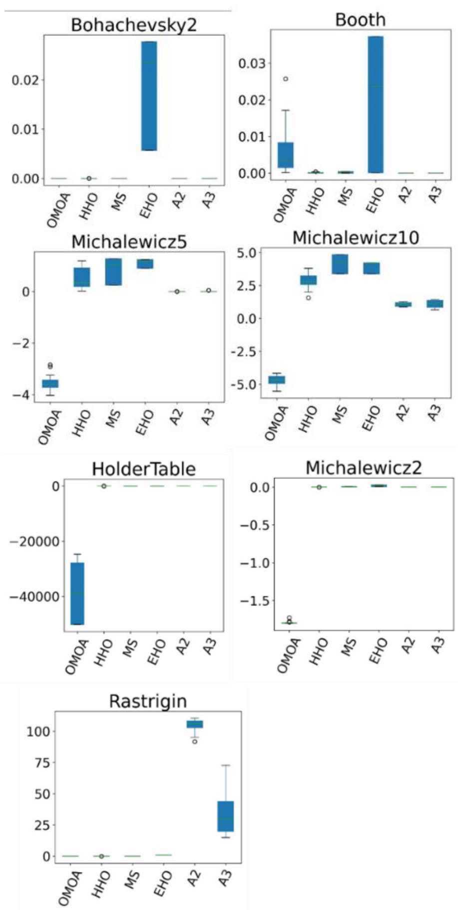


Fig. 14. Multimodal and separable boxplot.

The visuals of the boxplot in Fig. 14 show mean solutions of ỌMOA adequately located in the region with very few deviations and outliers.

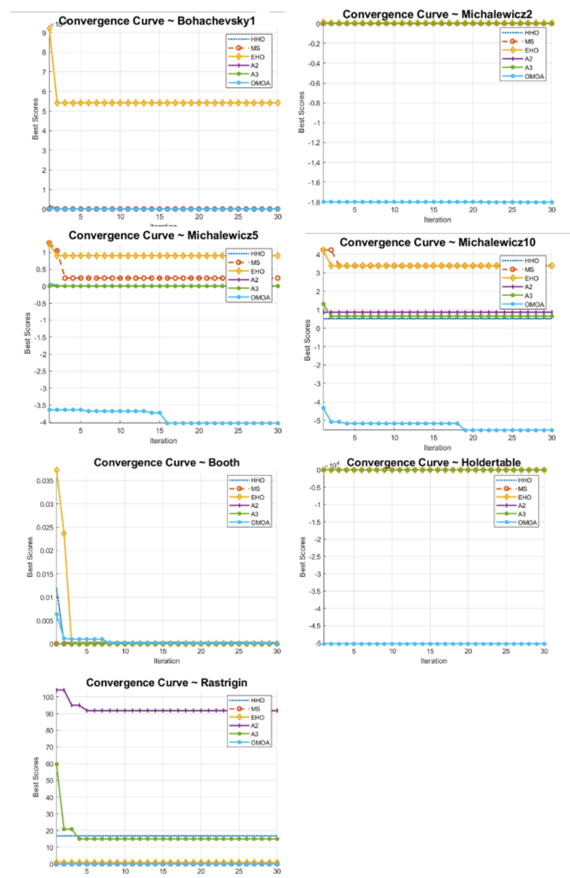


Fig. 15. Convergence curves for multimodal and separable function.

In Fig. 15, ỌMOA had made extra-advance to explore for solutions far better than all the compared algorithms in these problems. Even some of the solutions were far better optimum that set global values as the Holder Table model.

F. Multimodal and Non-Separable

Table VII shows ỌMOA led the exploration alongside A3, A2, and HHO though the depth of the troughs in Six-Hump Camel, Shubert, and Drop Wave seems to have shown that ỌMOA dived deeper than the others with the peeks of Schaffer 4 also.

TABLE VII. MULTIMODAL AND NON-SEPARABLE RESULTS

Function	Measure	ỌMOA	HHO	MS	EHO	A2	A3
F20	Best	**0.0E+00**	**0.0E+00**	**0.0E+00**	3.3E-07	**0.0E+00**	**0.0E+00**
n = 2	Worst	**4.3E-01**	**0.0E+00**	**0.0E+00**	6.6E-07	**0.0E+00**	**0.0E+00**
f = 0	Mean	**9.3E-02**	**0.0E+00**	**0.0E+00**	4.9E-07	**0.0E+00**	**0.0E+00**
	Sd	**1.2E-01**	**0.0E+00**	**0.0E+00**	5.1E-07	**0.0E+00**	**0.0E+00**
F21	Best	**0.0E+00**	**0.0E+00**	**5.8E-08**	1.3E-06	**1.0E-08**	**0.0E+00**
n = 4	Worst	**1.6E-02**	**1.1E-05**	**7.4E-06**	1.5E-04	**2.5E-05**	**0.0E+00**
f= 0.29259	Mean	**1.3E-03**	**1.2E-06**	**2.6E-06**	5.3E-05	**6.1E-06**	**0.0E+00**
	Sd	**3.7E-03**	**3.2E-06**	**4.3E-06**	8.7E-05	**9.3E-06**	**0.0E+00**
F22	Best	9.7E-03	0.0E+00	0.0E+00	5.2E+00	6.7E+00	**5.5E+00**
n = 6	Worst	4.0E-01	2.3E-03	2.2E-08	6.1E+00	9.1E+00	**7.5E+00**
f = 0	Mean	1.7E-01	9.0E-05	7.3E-09	5.7E+00	8.1E+00	**6.6E+00**
	Sd	1.2E-01	4.3E-04	1.3E-08	5.8E+00	8.1E+00	**6.7E+00**
F23	Best	**-1.0E+00**	0.0E+00	8.6E-07	1.7E-04	0.0E+00	0.0E+00
n = 2	Worst	**-1.0E+00**	1.7E-08	3.4E-05	1.9E-03	1.1E-07	0.0E+00
f= -1.03163	Mean	**-1.0E+00**	2.2E-09	1.6E-05	8.2E-04	3.2E-08	0.0E+00
	Sd	**9.6E-05**	5.5E-09	2.1E-05	1.1E-03	5.3E-08	0.0E+00
F24	Best	**0.0E+00**	**0.0E+00**	1.2E-08	5.7E-03	**0.0E+00**	**0.0E+00**
n = 2	Worst	**0.0E+00**	**3.3E-07**	2.1E-07	2.8E-02	**0.0E+00**	**0.0E+00**
f = 0	Mean	**0.0E+00**	**2.5E-08**	1.2E-07	1.9E-02	**0.0E+00**	**0.0E+00**
	Sd	**0.0E+00**	**7.7E-08**	1.4E-07	2.1E-02	**0.0E+00**	**0.0E+00**
F25	Best	**0.0E+00**	0.0E+00	0.0E+00	3.0E-03	**0.0E+00**	**0.0E+00**
n = 2	Worst	**0.0E+00**	9.0E-06	1.4E-08	9.5E-03	**0.0E+00**	**0.0E+00**
f = 0	Mean	**0.0E+00**	6.2E-07	4.6E-09	7.2E-03	**0.0E+00**	**0.0E+00**
	Sd	**0.0E+00**	1.9E-06	8.0E-09	7.8E-03	**0.0E+00**	**0.0E+00**
F26	Best	**-1.9E+02**	**-9.0E-06**	1.1E-02	2.3E-01	**-6.0E-06**	**-9.0E-06**
n = 2	Worst	**-1.9E+02**	2.7E-04	3.2E-02	1.8E+00	**8.2E-02**	**-9.0E-06**
f = -186.73	Mean	**-1.9E+02**	2.4E-05	2.2E-02	1.2E+00	**2.8E-02**	**-9.0E-06**
	Sd	**9.3E-02**	6.5E-05	2.3E-02	1.4E+00	**4.0E-02**	**0.0E+00**
F27	Best	**-1.0E+00**	0.0E+00	0.0E+00	3.9E-06	0.0E+00	0.0E+00
n = 2	Worst	**-1.0E+00**	0.0E+00	1.8E-07	3.8E-05	0.0E+00	0.0E+00
f = -1	Mean	**-1.0E+00**	0.0E+00	6.6E-08	2.6E-05	0.0E+00	0.0E+00
	Sd	**0.0E+00**	0.0E+00	1.1E-07	3.0E-05	0.0E+00	0.0E+00
F28	Best	2.9E+01	1.1E-03	2.9E+01	3.8E+01	2.7E+01	**0.0E+00**
n = 30	Worst	2.9E+01	2.5E-01	2.9E+01	4.0E+01	2.8E+01	4.0E+00
f = 0	Mean	2.9E+01	4.5E-02	2.9E+01	3.9E+01	2.8E+01	1.2E+00
	Sd	9.0E-02	7.1E-02	2.9E+01	3.9E+01	2.8E+01	2.2E+00
F29	Best	**0.0E+00**	0.0E+00	0.0E+00	9.3E-01	6.1E-04	**0.0E+00**
n = 30	Worst	**0.0E+00**	1.1E-06	2.2E-08	1.0E+00	2.2E-03	**0.0E+00**
f = 0	Mean	**0.0E+00**	6.9E-08	7.3E-09	9.7E-01	1.3E-03	**0.0E+00**
	Sd	**0.0E+00**	2.1E-07	1.3E-08	9.8E-01	1.4E-03	**0.0E+00**
F30	Best	**8.9E-16**	1.1E-06	1.7E-05	4.3E-01	3.9E-03	1.9E-06
n = 30	Worst	**8.9E-16**	1.0E-04	3.6E-05	5.4E-01	6.2E-03	1.7E+00
f = 0	Mean	**8.9E-16**	1.8E-05	2.4E-05	4.8E-01	5.0E-03	7.2E-01
	Sd	**8.9E-16**	2.7E-05	2.5E-05	4.9E-01	5.0E-03	9.0E-01

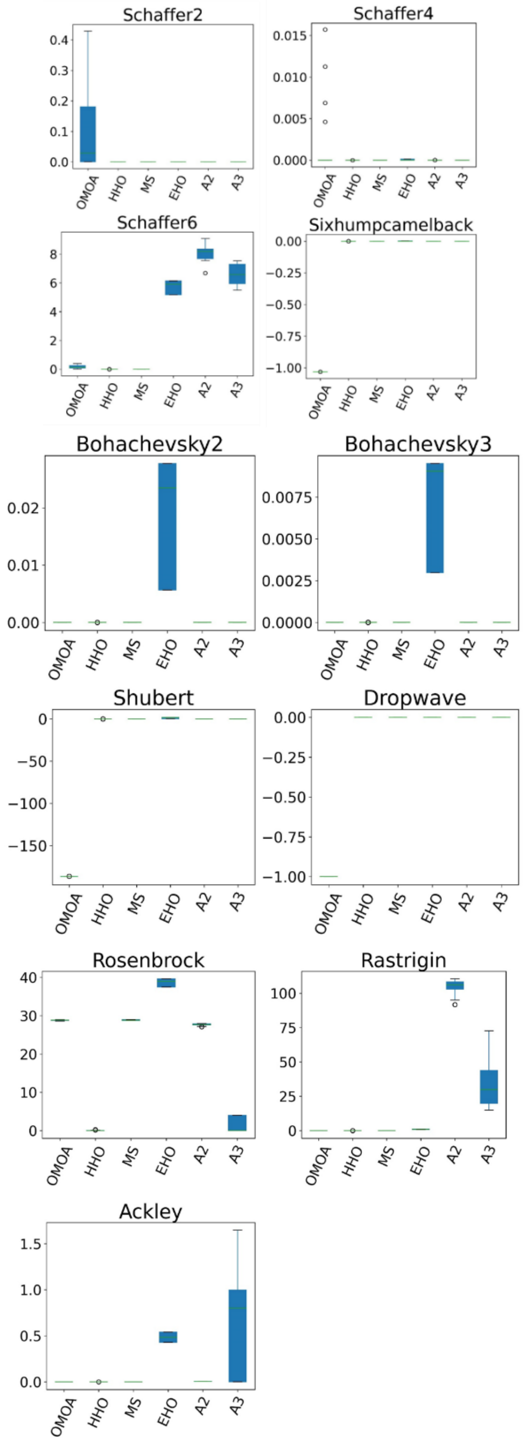


Fig. 16. Multimodal and non-separable boxplot.

The boxplot in Fig. 16 shows the clear visuals, confirms ỌMOA better. The exploratory ability of the ỌMOA is evident from mean solutions distribution and standard deviations.

The convergence is a reflection of the ability of the tree depth of the network of markets embedded in the model visualized in Fig. 17.

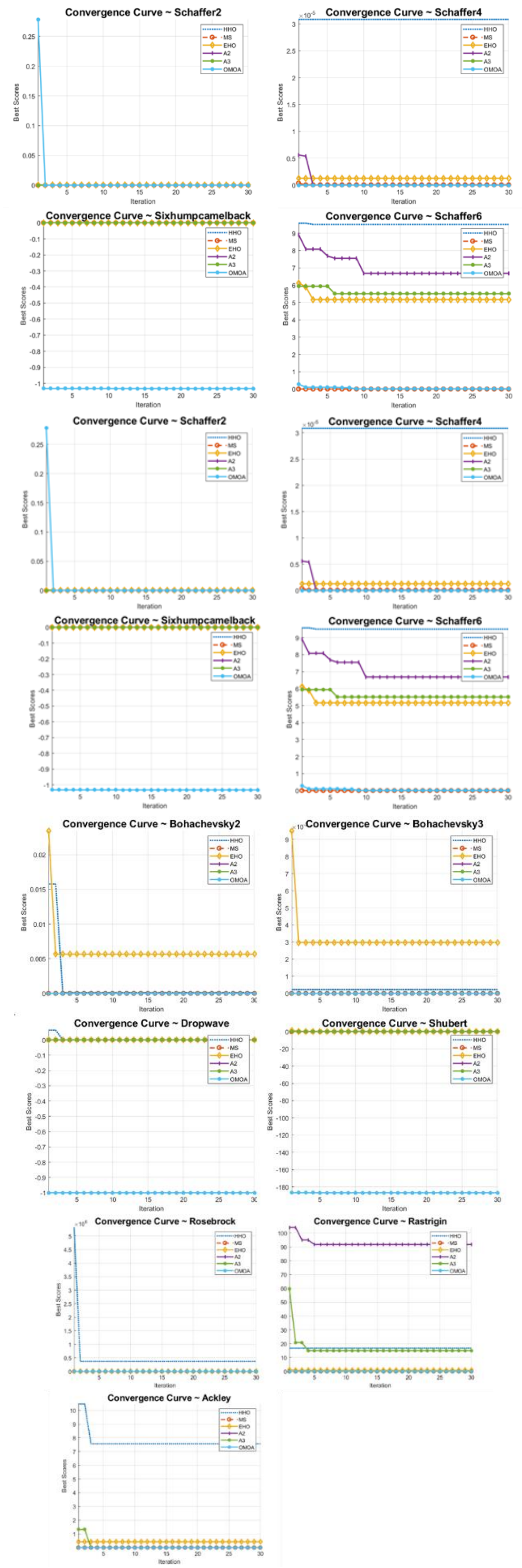


Fig. 17. Convergence curves for multimodal and non-separable function.

G. Statistical Test and Significance

Table VIII presents the entire statistical hypothesis test carried out to confirm the difference in mean and significance validation in the distribution of solutions by the algorithms on the 30 unconstraint benchmark functions.

Table IX is the summary of the test conducted to prove the hypothesis of the performances of the experiment; **1**: means ỌMOA (in black ink) is more significant, **-1**: gives better significance to the contender (another algorithm), while **0**: depicts no significant difference in performance (contender, equal, ỌMOA).

TABLE VIII. T-TEST FOR PERFORMANCE AND HYPOTHESIS

S/N	Algorithm/ Function	HHO			MS			EHO			A2			A3		
		t-value	p-value	Sign	t-value	p-value	Sign	t-value	p-value	Sign	t-value	p-value	Sign	t-value	p-value	Sign
1	**Step**	-5.8E+00	2.5E-06	**1**	-2.8E+01	2.1E-22	**1**	-7.5E+01	8.7E-35	**1**	-1.3E+01	8.9E-14	**1**	nan	nan	**0**
2	**Sphere**	-2.3E+00	3.1E-02	**1**	nan	nan	**0**	-1.4E+01	9.6E-15	**1**	-1.5E+01	5.4E-15	**1**	nan	nan	**0**
3	**Sum-Square**	-1.5E+00	1.4E-01	**1**	-7.5E+00	2.8E-08	**1**	-4.5E+01	2.4E-28	**1**	-1.4E+01	1.0E-14	**1**	nan	nan	**0**
4	**Quartic**	nan	nan	**0**	nan	nan	**0**	-2.0E+01	1.9E-18	**1**	-9.7E+00	1.4E-10	**1**	nan	nan	**0**
5	**Beale**	5.3E+00	9.9E-06	**-1**	5.3E+00	1.1E-05	**-1**	4.1E+00	3.3E-04	**-1**	5.3E+00	9.9E-06	**1**	5.3E+00	9.9E-06	**-1**
6	**Easom**	-4.6E+00	7.5E-05	**1**	-4.6E+00	7.2E-05	**1**	-8.0E+00	9.1E-09	**1**	-4.6E+00	7.5E-05	**1**	-4.6E+00	7.5E-05	**1**
7	**Matyas**	nan	nan	**0**	nan	nan	**0**	-9.4E+00	2.6E-10	**1**	nan	nan	**0**	nan	nan	**0**
8	**Colville**	-5.3E+00	1.1E-05	**1**	-5.1E+00	2.0E-05	**1**	-3.0E+01	1.7E-23	**1**	nan	nan	**0**	nan	nan	**0**
9	**Zakharov**	-2.7E+00	1.3E-02	**1**	-3.8E+00	6.7E-04	**1**	-1.4E+01	4.0E-14	**1**	-4.4E+00	1.3E-04	**1**	nan	nan	**0**
10	**Schwefel 2.22**	-4.2E+00	2.4E-04	**1**	-5.3E+00	1.1E-05	**1**	-4.8E+01	2.7E-29	**1**	-1.7E+01	2.7E-16	**1**	-9.9E+00	9.2E-11	**1**
11	**Schwefel 1.2**	-1.4E+00	1.6E-01	**0**	-1.1E+01	1.2E-11	**1**	-6.1E+01	3.0E-32	**1**	-1.1E+01	2.3E-11	**1**	nan	nan	**0**
12	**Dixon Price**	-3.6E+01	1.0E-25	**1**	-2.7E+02	1.3E-50	**1**	-8.1E+01	1.1E-35	**1**	-1.1E+04	7.8E-98	**1**	-3.0E+04	3.3E-110	**1**
13	**Bohachevsky 1**	-2.2E+00	3.3E-02	**1**	-9.1E+00	5.5E-10	**1**	-1.3E+01	9.3E-14	**1**	nan	nan	**0**	nan	nan	**0**
14	**Booth**	5.1E+00	2.1E-05	**-1**	4.9E+00	3.4E-05	**-1**	-5.5E+00	7.3E-06	**1**	5.2E+00	1.6E-05	**-1**	5.2E+00	1.6E-05	**-1**
15	**Holder Table**	-1.8E+01	2.1E-17	**1**	-1.8E+01	2.1E-17	**1**	-1.8E+01	2.1E-17	**1**	-1.8E+01	2.1E-17	**1**	-1.8E+01	2.1E-17	**1**
16	**Michalewicz 2**	-7.1E+02	5.9E-63	**1**	-6.8E+02	2.0E-62	**1**	1.4E-59	1.3E-53	**-1**	-7.1E+02	5.9E-63	**1**	-7.1E+02	5.9E-63	**1**
17	**Michalewicz 5**	-4.5E+01	1.9E-28	**1**	-4.8E+01	2.9E-29	**1**	-7.5E+01	9.8E-35	**1**	-6.9E+01	9.8E-34	**1**	-7.1E+01	4.4E-34	**1**
18	**Michalewicz 10**	-6.7E+01	2.9E-33	**1**	-6.6E+01	4.2E-33	**1**	-9.5E+01	9.2E-38	**1**	-8.0E+01	1.4E-35	**1**	-7.5E+01	1.0E-34	**1**
19	**Rastrigin**	-1.9E+00	7.1E-02	**0**	-6.9E+00	1.2E-07	**1**	-4.0E+01	5.0E-27	**1**	-9.5E+01	9.4E-38	**1**	-1.0E+01	3.2E-11	**1**
20	**Schaffer 2**	4.1E+00	3.1E-04	**-1**	4.1E+00	3.1E-04	**-1**	4.1E+00	3.1E-04	**-1**	4.1E+00	3.1E-04	**-1**	4.1E+00	3.1E-04	**-1**
21	**Schaffer 4**	1.4E+00	1.7E-01	**0**	1.9E+00	6.7E-02	**0**	1.8E+00	7.8E-02	**0**	1.9E+00	6.8E-02	**0**	1.9E+00	6.6E-02	**0**
22	**Schaffer 6**	7.8E+00	1.3E-08	**-1**	7.8E+00	1.2E-08	**-1**	-6.4E+01	9.9E-33	**1**	-6.6E+01	3.1E-33	**1**	-5.0E+01	8.9E-30	**1**
23	**6HumCamel Back**	-5.9E+04	1.0E-118	**1**	-6.1E+04	3.5E-119	**1**	-7.3E+03	2.6E-92	**1**	-5.9E+04	1.0E-118	**1**	-5.9E+04	1.0E-118	**1**
24	**Bohachevsky 2**	-1.8E+00	7.8E-02	**0**	-7.8E+00	1.3E-08	**1**	-1.1E+01	1.5E-11	**1**	nan	nan	**0**	nan	nan	**0**
25	**Bohachevsky 3**	-1.9E+00	7.0E-02	**0**	-7.8E+00	1.3E-08	**1**	-3.8E+00	6.7E-04	**1**	nan	nan	**0**	nan	nan	**0**
26	**Shubert**	-1.1E+04	1.5E-97	**1**	-1.1E+04	2.2E-97	**1**	-1.4E+03	8.3E-72	**1**	-1.0E+04	2.1E-96	**1**	-1.1E+04	1.5E-97	**1**
27	**Drop Wave**	-inf	0.0E+00	**1**	-6.6E+07	3.4E-207	**1**	-3.5E+05	5.8E-141	**1**	-inf	0.0E+00	**1**	-inf	0.0E+00	**1**
28	**Rosenbrock**	1.7E+03	9.7E-74	**-1**	-3.3E-02	9.7E-01	**0**	-5.9E+01	1.0E-31	**1**	2.2E+01	1.9E-19	**-1**	8.2E+01	7.4E-36	**-1**
29	**Grienwank**	-1.9E+00	7.4E-02	**0**	-3.8E+00	6.7E-04	**1**	-1.4E+02	2.2E-42	**1**	-1.3E+01	6.9E-14	**1**	nan	nan	**0**
30	**Ackley**	-4.9E+00	3.4E-05	**1**	-1.5E+01	7.1E-15	**1**	-5.7E+01	2.3E-31	**1**	-3.2E+01	3.8E-24	**1**	-7.2E+00	7.0E-08	**1**

Sign (better) ==> Significance (1 = ỌMOA; -1 = Alternative Algorithm; 0 = no significant difference), nan = not a number, inf = infinite

TABLE IX. SUMMARY OF SIGNIFICANCE AND RANK

(-1,0,1)	HHO	MS	EHO	A2	A3
ỌMOA	[5, 8, **17**]	[4, 5, **21**]	[2, 1, **27**]	[3, 6, **21**]	[4, 13, **13**]

The highest equal performance point, 13 is between ỌMOA and A3, with ỌMOA leading with 13 optimal solutions, more than A3's other 4 better performances. Next is HHO with 8 equal points, ỌMOA with 7 better optimal solutions, and HHO making 5 places. MS and A1 shared very close contest with EHO behind. Table X also is the presentation of the mean runtime measure given below.

TABLE X. RUNTIME TEST RESULTS BASED ON RASTRIGIN

ỌMOA	**HHO**	**MS**	**EHO**	**A2**	**A3**
2.493096	45.6716	756.4435	788.1535	202.0933	217.6285

The performance of ỌMOA in this section was very high on benchmark complex unconstraint problems compared to contending methods.

H. Results and Statistical Testing with CEC 2017

This section reports ỌMOA on real-parameter single objective optimization challenging problems featured in Computational Evolution Computation - CEC 2017 with a statistical comparison between ỌMOA and winners of the competition.

I. Result of ỌMOA with CEC 2017

The values are the differences between the global optima and the ones obtained with ỌMOA for 10D, 30D, and 100D during every 51 runs, as shown in the Table XI, and competition is presented in Table XII for 50D

1) 10D, 30D, 50D and 100D Performances

- The uni-modal functions EC1, EC2, and EC3 results were least expected within the number of functional evaluations provided, perhaps due to parameter tuning differences from recommended.
- EC7 – EC10 multimodal functions all attained global optima in all dimensions. ỌMOA also met 10D and 30D optimal values, with a minor difference for 50D and not too good 100D. EC5 solutions are not good in all dimensions; while EC6 10D was globally optimal, the rest dimensions were not impressive and inadequate for some ranges of solutions.
- Hybrid functions optimization; ỌMOA yielded optimal global solutions for EC11, EC14 – EC17, and EC19 - EC20 leaving out EC12, EC13, and EC18 with not too good in solutions.
- Besides EC21, EC22, and EC27 of the Composition functions with non-optimal solutions, ỌMOA achieved optimal global solutions for others, i.e., EC23, EC24, EC25, EC26, EC28, and EC29 in all dimensions, respectively.

J. Time Complexity Analysis

The competition provided appropriate information on the modalities to compute the time complexity [39]. The observation and experimentation shown in Table XII of this work is as follows:

- Evaluate a code consisting of basic arithmetic operation for 1,000,000 iterations and recode the time (T_0).
- Evaluate the hybrid function EC18 for 200,000 times the four dimensions (10D, 30D, 50D, and 100D) with record (T_1).
- For every dimension, find the meantime of computing $\bar{T}_2$ the hybrid function EC18 five times run with a termination iteration of 200,000.
- Calculate the time complexity of the algorithm using the relation $(\bar{T}_2 - T_1)/T_0$.

Table XIII depicts that the complexity of ỌMOA is not increasing significantly with the increase in the dimension of the functions.

TABLE XI. STATISTICAL RESULTS FOR CEC 2017 SIMULATIONS D10, D30, AND D100

Tag	Best			Worst			Mean			Median			Standard Deviation		
Tag	**10D**	**30D**	**100D**	**10D**	**30D**	**100D**	**10D**	**30D**	**100D**	**10D**	**30D**	**100D**	**10D**	**30D**	**100D**
EC 1	3.6E+3	6.3E+4	5.8E+5	1.7E+4	1.3E+5	1.7E+6	1.1E+4	9.9E+4	1.1E+6	1.1E+4	1.0E+5	1.1E+6	3.1E+3	1.6E+4	2.7E+5
EC 2	3.6E+3	6.3E+4	5.8E+05	1.7E+4	1.3E+05	1.7E+006	1.1E+4	9.9E+04	1.1E+006	1.1E+4	1.0E+05	1.1E+06	3.1E+3	1.6E+04	2.7E+05
EC 3	1.9E+03	7.3E+4	7.9E+5	1.8E+4	1.9E+5	1.8E+6	9.0E+3	1.3E+5	1.4E+6	8.9E+3	1.3E+5	1.4E+6	3.5E+3	3.1E+4	2.5E+5
EC 4	0.0E+0	0.0E+0	1.9E+3	0.0E+0	0.0E+0	2.5E+3	0.0E+0	0.0E+0	2.3E+3	0.0E+0	0.0E+0	2.3E+3	0.0E+0	0.0E+0	1.4E+2
EC 5	1.5E+7	4.0E+7	1.2E+8	1.5E+7	4.0E+7	1.2E+8	1.5E+7	4.0E+7	1.2E+8	1.5E+7	4.0E+7	1.2E+8	0.0E+0	0.0E+0	0.0E+0
EC 6	0.0E+0	4.4E+2	9.0E+3	0.0E+0	1.8E+3	1.2E+4	0.0E+0	1.3E+3	1.1E+4	0.0E+0	1.3E+3	1.1E+4	0.0E+0	2.9E+2	6.6E+2
EC 7	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0
EC 8	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0
EC 9	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0
EC 10	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0

EC 11	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0
EC 12	3.7E+3	4.5E+4	2.4E+5	1.4E+4	7.7E+4	3.2E+5	8.5E+3	6.2E+4	3.0E+5	8.4E+3	6.2E+4	3.0E+5	2.6E+3	6.5E+3	1.5E+4
EC 13	1.6E+8	1.4E+10	4.9E+10	3.7E+9	3.8E+10	9.4E+10	1.8E+9	2.8E+10	7.3E+10	1.8E+9	2.8E+10	7.4E+10	8.3E+8	5.6E+9	1.1E+10
EC 14	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0
EC 15	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0
EC 16	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0
EC 17	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0
EC 18	3.8E+3	4.6E+4	2.4E+5	1.4E+4	7.7E+4	3.2E+5	8.5E+3	6.2E+4	3.0E+5	8.3E+3	6.2E+4	3.0E+5	2.6E+3	6.5E+3	1.5E+4
EC 19	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0
EC 20	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0
EC 21	1.3E+4	1.6E+5	7.9E+5	4.7E+4	2.6E+5	1.1E+6	2.8E+4	2.2E+5	1.0E+6	2.9E+4	2.2E+5	1.0E+6	8.0E+3	2.6E+4	6.8E+4
EC 22	3.6E+9	2.1E+11	1.8E+12	4.0E+10	6.1E+11	3.3E+12	1.6E+10	4.0E+11	2.7E+12	1.5E+10	3.9E+11	2.7E+12	7.8E+9	9.8E+10	3.3E+11
EC 23	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0
EC 24	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0
EC 25	0.0E+0	0.0E+0	5.0E+3	0.0E+0	0.0E+0	6.1E+3	0.0E+0	0.0E+0	5.6E+3	0.0E+0	0.0E+0	5.7E+3	0.0E+0	0.0E+0	2.9E+2
EC 26	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0
EC 27	1.3E+4	1.6E+5	7.9E+5	4.7E+4	2.6E+5	1.1E+6	2.8E+4	2.2E+5	1.0E+6	2.9E+4	2.2E+5	1.0E+6	8.0E+3	2.6E+4	6.8E+4
EC 28	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0
EC 29	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0	0.0E+0

TABLE XII. STATISTICAL COMPARISON OF ỌMOA AND STATE-OF-THE-ART ALGORITHMS FOR CEC 2017, 50D

Tag	JADE	SHADE	UMOEAsII	MVMO	LSHADE-*cn*EpSin	EBOwithCMAR	ỌMOA
EC 1	**5.2385E-14 (2.5180E-14)** +	**0.0000E+00 (0.0000E+00)** +	**0.0000E+00 (0.0000E+00)** +	**1.3313E-05 (5.6019E-06)** +	**0.0000E+00 (0.0000E+00)** +	**0.00+00 (0.00+00)** +	2.80E+05 (6.00E+04)
EC 2	1.3112E+13 (8.5354E+13) –	1.0801E+12 (4.3906E+12) –	**0.0000E+00 (0.0000E+00)** +	1.8060E+17 (1.2778E+18) –	**1.5686E+00 (1.9314E+00)** +	**0.00+00 (0.00+00)** +	2.20E+05 (3.60E+04)
EC 3	**1.7712E+04 (3.7017E+04)** +	**0.0000E+00 (0.0000E+00)** +	**2.1202E-09 (8.8715E-09)** +	**5.3095E-07 (1.0965E-07)** +	**0.0000E+00 (0.0000E+00)** +	**0.00+00 (0.00+00)** +	3.40E+05 (6.60E+04)
EC 4	4.9625E+01 (4.7914E+01) -	5.6885E+01 (4.6262E+01) –	6.5462E+01 (5.2164E+01) –	3.5808E+01 (3.6684E+01) -	5.1401E+01 (4.4262E+01) -	4.29E+0l (3.32E+0l) -	**3.30E+01 (4.80E+01)**
EC 5	**5.4288E+01 (8.8034E+00)** +	**3.2859E+01 (5.0387E+00)** +	**5.0801E+00 (1.6684E+00)** +	**8.0787E+01 (1.6432E+01)** +	**2.5166E+01 (6.4447E+00)** +	**7.58E+00 2.42E+00** +	6.60E+07 (0.00E+00)
EC 6	**1.4489E-13 (9.1172E-14)** +	**8.3876E-04 (1.0169E-03)** +	**1.1951E-06 (1.9013E-06)** +	**5.4321E-03 (3.3038E-03)** +	**9.1569E-07 (1.0750E-06)** +	**8.54E-08 (1.14E-07)** +	4.00E+03 (3.80E+02)
EC 7	1.0140E+02 (6.4883E+00) –	8.0964E+01 (3.7800E+00) –	5.6459E+01 (7.1546E-01) -	1.2320E+02 (1.2795E+01) -	7.6639E+01 (6.0618E+00) -	5.79E+01 (1.53E+00) -	**0.00E+00 (0.00E+00)**
EC 8	5.5234E+01 (7.7643E+00) –	3.2355E+01 (3.8252E+00) –	4.7781E+00 (1.6264E+00) -	7.5910E+01 (1.6122E+01) –	2.6319E+01 (6.5917E+00) -	7.9IE+00 (2.47E+00) -	**0.00E+00 (0.00E+00)**
EC 9	1.1773E+00 (1.3141E+00) –	1.1123E+00 (9.3715E-01) –	1.7555E-03 (1.2536E-02) –	7.3843E+00 (5.7735E+00) –	**0.0000E+00 (0.0000E+00)** =	**0.00+00 (0.00+00)** =	**0.00E+00 (0.00E+00)**
EC 10	3.7500E+03 (2.5448E+02) –	3.3444E+03 (2.9402E+02) –	3.3804E+03 (4.7255E+02) –	3.4971E+03 (4.3138E+02) –	3.2001E+03 (3.3972E+02) -	3. 11E+03 (4.0lE+02) -	**0.00E+00 (0.00E+00)**
EC 11	1.3612E+02 (3.3972E+01) –	1.2065E+02 (2.9317E+01) –	4.5701E+01 (9.1852E+00) –	4.7488E+01 (8.7237E+00) –	2.1393E+01 (2.0902E+00) -	2.64E+01 (3.36E+00) -	**0.00E+00 (0.00E+00)**
EC 12	**5.1468E+03 (3.3233E+03)** +	**5.1362E+03 (2.8785E+03)** +	**2.1449E+03 (5.3559E+02)** +	**1.2955E+03 (2.7935E+02)** +	**1.4753E+03 (3.6472E+02)** +	**1.94E+03 (5.34E+02)** +	1.20E+05 (1.00E+04)

EC 13	**3.0338E+02 (2.6999E+02) +**	**2.6565E+02 (1.4944E+02) +**	**5.1787E+01 (2.1985E+01) +**	**4.3776E+01 (1.7622E+01) +**	**6.9430E+01 (3.4457E+01) +**	**4.14E+01 (2.45E+01) +**	6.90E+10 (1.10E+10)
EC 14	1.0519E+04 (3.1138E+04) –	2.1578E+02 (7.2995E+01) –	2.9299E+01 (2.4831E+00) -	4.8524E+01 (1.2153E+01) –	2.6522E+01 (2.4924E+00) -	3.12E+0I (3.52E+00) -	**0.00E+00 (0.00E+00)**
EC 15	3.4992E+02 (4.4266E+02) –	3.2262E+02 (1.4201E+02) –	4.1468E+01 (1.0651E+01) –	4.4630E+01 (1.1280E+01) –	2.5596E+01 (4.0567E+00) -	2.94E+0I (5.20E+00) -	**0.00E+00 (0.00E+00)**
EC 16	8.5696E+02 (1.7532E+02) –	7.3389E+02 (1.8854E+02) –	3.9288E+02 (1.5514E+02) –	8.4082E+02 (1.9349E+02) –	2.7453E+02 (9.9692E+01) -	3.46E+02 (1.46E+02) -	**0.00E+00 (0.00E+00)**
EC 17	6.0010E+02 (1.2128E+02) –	5.1634E+02 (1.1109E+02) –	3.1356E+02 (1.0636E+02) –	5.1999E+02 (1.3382E+02) –	2.0706E+02 (7.3064E+01) -	2.75E+02 (5.63E+0I) -	**0.00E+00 (0.00E+00)**
EC 18	**1.8906E+02 (1.2561E+02) +**	**1.8946E+02 (1.0338E+02) +**	**3.5997E+01 (8.7118E+00) +**	**4.1756E+01 (1.9445E+01) +**	**2.4332E+01 (2.1179E+00) +**	**3.20E+01 (5.99E+00) +**	1.20E+05 (1.00E+04)
EC 19	3.2429E+02 (1.2561E+03) –	1.5976E+02 (5.6842E+01) –	2.2807E+01 (3.7669E+00) –	1.7338E+01 (5.1321E+00) -	1.7406E+01 (2.4713E+00) -	2.45E+0I (3.94E+00) -	**0.00E+00 (0.00E+00)**
EC 20	4.3806E+02 (1.3382E+02) –	3.3382E+02 (1.2079E+02) –	2.3041E+02 (1.2312E+02) –	3.2965E+02 (1.4772E+02) –	1.1412E+02 (3.5483E+01) -	1.47E+02 (7.44E+0l) -	**0.00E+00 (0.00E+00)**
EC 21	**2.5198E+02 (9.6384E+00) +**	**2.3338E+02 (5.1139E+00) +**	**2.0681E+02 (2.5498E+00) +**	**2.7719E+02 (1.6036E+01) +**	**2.2676E+02 (7.0598E+00) +**	**2. 11E+02 (4.06E+00) +**	4.30E+05 (3.40E+04)
EC 22	**3.3364E+03 (1.8053E+03) +**	**3.1774E+03 (1.5566E+03) +**	**1.7929E+03 (1.9112E+03) +**	**3.2653E+03 (1.7185E+03) +**	**1.5950E+03 (1.6659E+03) +**	**3.65E+02 (9.24E+02) +**	9.20E+11 (1.70E+11)
EC 23	4.7956E+02 (1.1766E+01) –	4.5916E+02 (8.7508E+00) –	4.3459E+02 (5.2143E+00) -	5.0490E+02 (1.5646E+01) –	4.3929E+02 (6.9001E+00) -	4.34E+02 8.16E+00 -	**0.00E+00 (0.00E+00)**
EC 24	5.4197E+02 (7.6206E+00) –	5.3106E+02 (7.4577E+00) –	5.0810E+02 (2.6001E+00) -	5.8374E+02 (1.6940E+01) –	5.1282E+02 (5.5948E+00) -	5.06E+02 (3.85E+00) -	**0.00E+00 (0.00E+00)**
EC 25	5.1923E+02 (3.4820E+01) –	5.0694E+02 (3.6446E+01) –	4.8281E+02 (6.4445E+00) –	5.0912E+02 (3.1226E+01) –	4.8034E+02 (1.0816E+00) -	4.89E+02 (2.47E+0l) -	**0.00E+00 (0.00E+00)**
EC 26	1.6146E+03 (1.2169E+02) –	1.4168E+03 (9.7281E+01) –	5.7211E+02 (4.0709E+02) -	1.9319E+03 (2.8632E+02) –	1.2026E+03 (1.1870E+02) -	7.06E+02 (4.06E+02) -	**0.00E+00 (0.00E+00)**
F27	**5.5080E+02 (2.3427E+01) +**	**5.4925E+02 (2.7842E+01) +**	**5.3743E+02 (1.7376E+01) +**	**5.4355E+02 (1.7557E+01) +**	**5.2543E+02 (9.2143E+00) +**	**5.22E+02 (7.75E+00) +**	4.30E+05 (3.40E+04)
EC 28	4.9185E+02 (2.0882E+01) –	4.7943E+02 (2.4173E+01) –	4.7289E+02 (2.1643E+01) –	4.6481E+02 (1.5047E+01) –	4.5913E+02 (1.1904E+01) -	4.67E+02 (1.79E+0I) -	**0.00E+00 (0.00E+00)**
EC 29	4.7761E+02 (8.0661E+01) –	4.8716E+02 (1.0502E+02) –	3.6326E+02 (2.0650E+01) –	4.8938E+02 (1.1489E+02) –	3.5289E+02 (9.7796E+00) -	3.47E+02 (1.97E+0I) -	**0.00E+00 (0.00E+00)**
w/t/l	**10/0/19**	**10/0/19**	**11/0/18**	**9/0/20**	**11/1/17**	**11/1/17**	

TABLE XIII. TIME COMPLEXITY ANALYSIS

Time Complexity	T1	T2	TO	(avT2 - T1)/T0
10D	0.164781	0.556989	6.80E-02	5.77E+00
30D	0.1695	0.627578	6.80E-02	6.74E+00
50D	0.18455	0.715318	6.80E-02	7.81E+00
100D	0.24258	0.846715	6.80E-02	8.89E+00

K. Comparison of ỌMOA with the Winners of CEC2017 (EC1-EC29)

The subsection presents a performance comparison between the 50D problem size for ỌMOA and other state-of-art high-performing algorithms, especially those that won the CEC2017 competition for real-parameter single objective optimization challenges, as shown in Table XII. The last row represents the values of a Wilcoxon rank-sum test at an alpha value of 0.05. The terms designate the status of the ỌMOA against each competing algorithm such that $w(+..mean..win)/t(=..mean..tie)/l(-..mean..loss)$ For an algorithm making its first entry, the results show very high success [41] shown by Table XII; ỌMOA had remarkably shown better performance on most of the complex problems considered, as the last row shows. Also, ỌMOA showed better performances compared with the winners of the competition (competing method, equity, ỌMOA ➔**w/t/l**), e.g., (EBOwithCMAR won 11, equal in 1 and ỌMOA won 17).

L. Benchmark Design Real Engineering CEC 2020 Single Objective Problems

Eight (8) difficult engineering design-constrained problems that exhibit functional inequality and equality constraints are considered; compared with state-of-the-art algorithms from CEC 2020 real-world optimization issues presented in [42-44]. Among the results presented are the experiments' statistical best, mean, median, worst, and standard deviations. Generally, all models follow a structure as shown in Eq. (15).

$$\min f(x)$$

$$s.t.: g_n(x) \le 0,\ n = 1, \ldots, m \tag{11}$$

Where f is the fitness, xs' are the design variables, g is the constraint with less than equality (often greater than for maximization problems), and n is the number of constraints. The conversion of the functional constraint from inequality to equality transforms the problem into equation (16).

$$f_p(x) = f(x) + o \sum_{n=1}^{m} \Phi_n \left[g_n(x)\right]^2$$

$$s.t. \quad o > 0 (i.e. penalty\ factor)$$

$$\Phi_n = \begin{cases} 1\ if\ g_n\ is\ violated \\ 0\ if\ g_n\ is\ satisfied \end{cases} \tag{12}$$

Where $f_{p\ (x)}$ is the penalized objective function. High-performing state-of-the-art algorithms are adopted and compared against the design of certain engineering problems of Fig. 18 (a: Welded beam), (b: Pressure Vessel, and c: Compression Spring). The constraint violations are considered, and the penalty function method is used, which often transforms a constrained problem into an unconstrained continuous counterpart for ease of implementation.

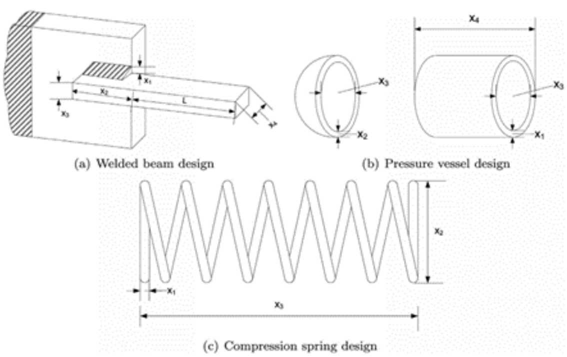


Fig. 18. Engineering design parameter problems.

M. Statistical Comparison of Results for Tension / Compression Spring Design Problem

The design problem in Fig 18 (c) aims at reducing the weight of the tension/compression spring without compromising domain properties like the shear stress, frequency wave, and displacement functionalities [45]. The control variables are wire diameter (x1), mean coil diameter (x2), and the number of coils (x3); the mathematical formulation is detailed in [42]. Upon the experiment, ỌMOA yielded the most optimal weight compared to the other high-performing algorithms within a minimal number of function evaluations. The result of the compared simulation is shown in Table XIV.

N. Statistical Comparison of the Results for Welded Beam Problem

The welded beam problem Fig. 18 (b) [46] is to minimize the cost of construction. The impacting constraints include shear stress(τ); bending stress in the beam (σ); buckling load of the bar (P_c); end deflection of the beam (δ) and side constraints. The decision variables are (1) the thickness of the weld (x1), the length of the attached part of bar (x2), the height of the bar (x3) and the thickness of the bar (x4). The model formulation is given in [42]. And compared simulated statistical results in Table XV with parametric results in Table XVI.

TABLE XIV. RESULTS FOR THE TENSION / COMPRESSION SPRING DESIGN PROBLEM

Method	Worst	Mean	Best	SD	NFEs
GA1	0,012822	0.012769	0.012704	3.94E − 05	900,000
GA2	0.012973	0.012742	0.012681	5.90E-05	80000
CAEP	0. 015116	0. 013568	0. 012721	8. 42E-04	50,020
CPSO	0.012924	0.012924	0.012674	5.20E-04	240,000
HPSO	0.012719	0.012707	0.012665	1.58E − 05	81,000
NM–PSO	0.012633	0.012631	0.01263	8.47E − 07	80,000
G-QPSO	0.017759	0.013524	0.012665	0.001268	2000
QPSO	0.018127	0.013854	0.012669	0.001341	2000
PSO	0.071802	0.019555	0.012857	0.011662	2000
DE	0.01279	0.012703	0.01267	2.7E − 05	204,800
DELC	0.012665	0.012665	0.012665	1.3E − 07	20,000
DEDS	0.012738	0.012669	0.012665	1.3E − 05	24,000
HEAA	0.012665	0.012665	0.012665	1.4E − 09	24,000
PSO–DE	0.012665	0.012665	0.012665	1.2E − 08	24,950
SC	0.016717	0.012922	0.012669	5.9E − 04	25,167
(μ + λ)-ES	NA	0.013165	0.012689	3.9E − 04	30,000
ABC	NA	0.012709	0.012665	1.28E − 02	30,000
LCA	0.01266667	0.0126654	0.0126652	3.88E − 07	15,000
WCA	0.012952	0.012746	0.012665	8.06E − 05	11,750
IGMM	0.0135125	0.0128657	0.0126652	2.56E − 04	4000
APSO	0.014937	0.013297	0.0127	6.85E − 04	120,000
MCEO	0.01350901	0.0127196	0.0126605	3.79E − 05	2000
OMOA	**0.01160**	**0.011241**	**0.0111090**	**0.0002354**	**2000**

TABLE XV. STATISTICAL RESULTS FOR WELDED BEAM PROBLEM

Method	Worst	Mean	Best	SD	NFEs
CAEP	3.179709	1.971809	1.724852	0.443	50,020
CPSO	1.782143	1.748831	1.7314849	0.0129	240,000
HPSO	1.814295	1.74904	1.724852	0.0401	81,000
PSO–DE	1.724852	1.724852	1.724852	6.7E − 16	66,600
NM–PSO	1.733393	1.726373	1.72472	0.0035	80,000
SC	6.399678	3.002588	2.385434	0.96	33,095
DE	1.824105	1.768158	1.733461	0.0221	204,800
WCA	1.744697	1.726427	1.724856	0.00429	46,450
LCA	1.7248523	1.7248523	1.7248523	7.11E − 15	15,000
IGMM	1.74769	1.732152	1.724855	7.14E − 03	8000
APSO	1.993999	1.877851	1.736193	0.076118	50,000
MCEO	1.7248732	1.7248621	1.7248523	1.02E − 05	12,500
ỌMOA	**1.6764577**	**1.622595**	**1.3534549**	**0.2390773**	**60000**

TABLE XVI. COMPARISONS OF THE BEST SOLUTIONS FOR WELDED BEAM

DV	CAEP	HGA	NM-PSO	WCA	IGMM	MCEO	ỌMOA
$x1$	0.2057	0.2057	0.20583	0.205728	0.205729	0.2057296	**0.15203**
$x2$	3.4705	3.4705	3.468338	3.470522	3.470496	3.4704887	**8.02626**
$x3$	9.0366	9.0366	9.036624	9.03662	9.036625	9.0366239	**6.08668**
$x4$	0.2057	0.2057	0.20573	0.205729	0.205730	0.2057296	**0.15775**
g1(x)	−769.34	−769.34	−770.3698	−0.034128	−771.187	-771.2021	**-0.3760+E4**
g2(x)	*4.48154*	*4.48154*	−0.053122	−3.49E − 05	−0.05976	-2.88E – 05	**-1.83846+E4**
g3(x)	−0.2283	−0.2283	−0.228310	−1.19e − 6	−0.228310	-0.228310	**-0.00002+E4**
g4(x)	0	0	*1.00E − 04*	−3.43298	−1e − 6	0	**0**
g5(x)	*2.60337*	*2.603347*	−0.031555	−0.080728	−0.0319920	-1.85E – 05	**-0.0222+E4**
g6(x)	−0.0807	−0.08070	−0.080830	−0.23554	−0.08072	-0.08073	**0**
g7(x)	−3.4332	−3.43321	−3.43316	−0.013503	−3.4329802	-3.43298	**-0.0003+E4**
$f(x)$	1.72457	1.724577	1.724720	1.724856	1.7248552	1.724852	**1.622595**

In less than 60000 functional evaluations of 10 runs, ỌMOA yielded a mean cost that is the most optimal for the welded beam in comparison while obeying the constraints.

O. *Results for Pressure Vessel Design Problem*

The Pressure Vessel Design objective in Fig. 18 (b) [47] is to minimize the cost associated with materials, building, and welding of a cylindrical vessel with capped ends and a hemispherical-shaped head. The impacting variables include the shell thickness x(1), the head thickness x(2), the inner radius x(3), and the length of the cylindrical section excluding the head x(4); the model formulation is given in [42].

The yield of ỌMOA on the Pressure Vessel design problem produced the best optimal mean value and had a far smaller number of function evaluations of Table XVII.

ỌMOA met all the inequality constraints; best mean fitness as shown, followed by MCEO, WCA, and NM-PSO, respectively. However, contrary to the large number assigned to the penalty using the other algorithms, ỌMOA found better results with negligible penalty value for problems of spring and welding beam, and even no penalty was applied to vessel design.

TABLE XVII. THE PRESSURE VESSEL DESIGN PROBLEM RESULTS

Method	Worst	Mean	Best	SD	NFEs
GA1	6308.497	6293.8432	6288.7445	7.4133	900,000
GA2	6469.322	6177.2533	6059.9463	130.9297	80,000
CPSO	6363.8041	6147.1332	6061.0777	86.45	240,000
HPSO	6288.677	6099.9323	6059.7143	86.2	81,000
NM–PSO	5960.0557	5946.7901	5930.3137	9.161	80,000
G-QPSO	7544.4925	6440.3786	6059.7208	448.4711	8000
QPSO	8017.2816	6440.3786	6059.7209	479.2671	8000
PSO	14076.324	8756.6803	6693.7212	1492.567	8000
CDE	6371.0455	6085.2303	6059.734	43.013	204,800
WCA	6590.2129	6198.6172	5885.3327	213.049	27,500
LCA	6090.6114	6070.5884	6059.8553	11.37534	24,000
IGMM	6061.2868	6060.1598	6059.7143	0.5421	8000
APSO	7544.49272	6470.71568	6059.7242	326.9688	200,000
MCEO	6060.3096	6060.0315	6059.7143	1.2532	7500
ỌMOA	**870.8983**	**848.7333**	**846.27055**	**7.7880052**	**2000**

P. *Robot Gripper Problem*

The complexity involved in manipulating the grippers to minimize the difference between the minimum and maximum forces of the robotic action is ongoing research. Seven design variables, geometric properties, with about seven inequality constraints, are targeted. The Mathematical formulations are found here in [48].

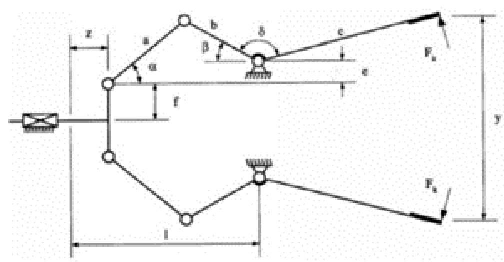


Fig. 19. Schematics of robotics gripper system.

Fig. 19 is schematics of the robotic gripper system, and the experimental result is shown in Table XVIII.

TABLE XVIII. THE STATISTICAL RESULTS OF ROBOTIC GRIPPER OPTIMIZATIONS

Variables	(TLBO)	AOS	ỌMOA
Best	4.247643634	2.54383686	**0.0000000000**
Mean	4.93770095	2.791745357	**0.6661765869**
Worst	8.141973	3.14335667	6.4675702717
Std-Dev	0.56	0.226323642	1.8645193071
a	150	149.9973899	27.248574186
b	150	149.880236	150
c	200	200	200
d	0	0	0
e	150	149.9954554	71.18076298
f	100	100.9429469	300
t	2.339539113	2.297394124	2.124666881
$g_1(x)$	-28.09283911	-49.99996461	-47.7172
$g_2(x)$	-21.90716089	-5.23E-06	-2.2828
$g_3(x)$	-33.64959994	-49.99996461	-200.6371
$g_4(x)$	-16.35040006	-3.53E-05	150.6371
$g_5(x)$	-79999.998	-79737.112	0
$g_6(x)$	-9.8E-11	-36.02117726	-4.0000
$g_7(x)$	-0.00001	-0.943046876	-0.0200

The experimental result of ỌMOA on the gripper problem showcases a new optimum as against the optimum global set value [42]; also better than the competing algorithms in comparison [44].

Q. *Rolling Element Bearing*

Five design variables that affect the optimal design of a rolling bearing with the capacity to carry load efficiently amidst nine inequality constraints are considered in the design. The mathematical derivations are provided by [42], while we show the schematics in Fig. 20.

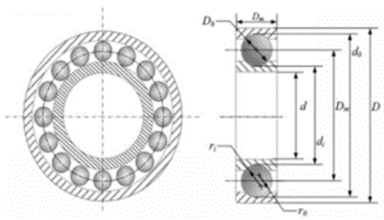


Fig. 20. Schematics of rolling bearing.

The result of the experiment is shown in Table XIX.

TABLE XIX. RESULTS OF ROLLING BEARING AND STATISTIC COMPARISONS WITH ỌMOA

	(TLBO)	ABC	GWO	ALO	AOS	ỌMOA
Best	81859.74	85428.2495	85529.0830	85546.6377	83918.49293	**6232.017129**
Mean	81438.987	85121.7544	83395.0849	84032.8636	82175.21266	**9468.313394**
Worst	80807.8551	83859.0851	43543.4508	73872.8164	83826.38337	14246.41460
Std-Dev	0.66	362.57	8224.5	3121.8	23.38511	2239.443627
D_m	21.42559	125.6599	125.7090	125.718	125	150
D_b	125.7191	21.40862	21.42316	21.425242	21.875	10.86042772
Z	11	11	11	11	10.77700905	4.510000000
f_i	0.515	0.515	0.515	0.515	0.515	0.594295410
f_0	0.515	0.515	0.529322	0.51517018	0.515	0.580231789
K_{Dmin}	0.424266	0.427166	0.420867	0,4541646	0.476110618	0.400000000
K_{Dmax}	0.633948	0.668849	0.633296	0.6464924	0.658142645	0.600000000
E	0.3	0.3	0.300224	0.30000122	0.3	0.300000000
e	0.068858	0.071386	0.02	0.0638003	0.02	0.020000000
Chi	0.799498	0.6	0.619432	0.6107592	0.618242202	0.600000000

With an optimum global set at (25287.918415), the experimental result of Table XIX shows that ỌMOA had set a better global optimum as it also performed better than the competing algorithms [44].

R. *Gas Transmission Compressor Design (GTCD)*

Four variables with one inequality constraint are targeted when designing the gas transmission compressor. The work [42] provides the mathematical formulation while we show the schematics in Fig. 21 and the solutions provided by many optimization state-of-the-art to designs.

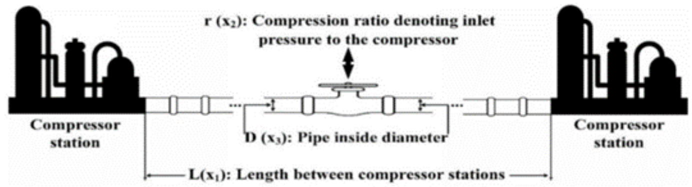


Fig. 21. Schematic of gas transmission compressor system with design variables.

The results of the comparison for the experiment on GTCD are shown in Table XX.

TABLE XX. RESULTS OF OPTIMIZATION OF GTCD AND STATISTICAL OUTCOMES

Algorithms					Optimum cost
	x1	x2	x3	x4	
CLPSO	45.8830	1.571778	27.18201	1.45592	3.7381430E+06
ABC	50.0000	1.185882	24.89145	0.39507	2.9845610E+06
ACOR	49.6067	1.174456	23.92940	0.37862	2.9671090E+06
ABC	50.0000	1.207839	24.49319	0.45792	2.9755610E+06
KH	35.6206	1.092393	31.99460	1.10937	3.4608480E+06
WOA	49.7095	1.178115	24.72718	0.38796	2.9650350E+06
HHO	49.9844	1.180801	24.20547	0.39429	2.9650910E+06
BOA	20.0000	1.000000	20.00000	0.16475	3.1364520E+06
HGSO	50.0000	1.164785	25.72731	0.35606	2.9689110E+06
LIACOR	50.0000	1.178480	24.58628	0.38882	2.9648960E+06
SMO	50.0000	1.178284	24.59259	0.38835	2.9648954E+06
OMOA	50.0000	1.00000	20.1422	60	**9.8081911E+05**

The experimental results show ỌMOA had a set a new global optimum than that set by the competition as the global optimum is (2.9648954173E+06) [42], with the other algorithms as presented in [49].

S. *Himmelblau's Function*

This nonlinear function has been used to test many novel metaheuristic algorithms; it has five main design variables and six inequality constraints to be handled, as shown in Himmelblau [50]. In Table XXI, we show the results of the performances of the metaheuristic algorithms used in comparison.

The experimental result shows that ỌMOA obtained a better minimum compared to the competing algorithms and set a new global optimum compared to the global presented by [42], which is −3.066554E+04, with the other algorithms as presented in [49].

T. *Multiple Disk Clutch Brake Design Problem*

The design objective is to minimize the mass of the multiple disk clutch brake, five decision variables with nine nonlinear constraints. The mathematical formulation is given in [42].

TABLE XXI. STATISTICAL AND PERFORMANCE OF ALGORITHMS ON THE HIMMELBLAU COMPLEX PROBLEM

Algorithm	X1	X2	X3		X4	**OPTIMAL**
CLPSO	86.3511	34.276	31.279		32.758	**−2.99E+04**
ABC	78	33.2729	30.65216	44.30402	36.4902	**−3.05E+04**
ACOR	78	33	30.04808	44.93806	36.7053	**−3.06E+04**
ABC	78	33	30.19617	45	36.3524	**−3.06E+04**
KH	78.99892	33.0057	30.67021	43.63579	35.5313	**−3.04E+04**
WOA	79.36031	33	30.04906	42.54110	37.2748	**−3.05E+04**
HHO	78	33	30.00757	44.99297	36.7473	**−3.06E+04**
BOA	78	33	30.31139	39.59049	31.5837	**−3.01E+04**
HGSO	78	33	3.109831	4.002299	3.62353	**−3.03E+04**
LIACOR	78	33	29.99526	45.00000	36.77581	−3.06E+04
SMO	78	33	29.995		36.7758	−3.06E+04
ỌMOA	79.8729	43.856	27.078	29.1039	29.1039	**-3.19E+04**

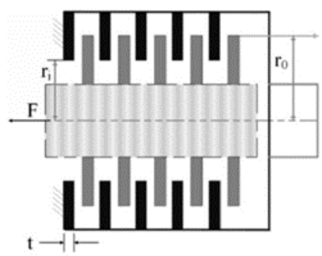


Fig. 22. The schematic geometric representation of the multiple disc clutch design.

Fig. 22 is schematics geometric representation of the clutch disc problem. However, the experimental results are shown in Table XXII.

TABLE XXII. SHOWS THE RESULTS OF THE PERFORMANCE OF METAHEURISTIC METHODS ON THE CLUTCH DESIGN PROBLEM

Algorithms	**x1**	**x2**	**x3**	**x4**	**x5**	**optimum**
CLPSO	75.95932	97.06936	1.01058	909.47864	2.09723	0.280152637613733
ABC	69.99974	90.00000	1.00000	697.47983	2.00000	0.235242474598156
ACOR	70.00000	90.00000	1.00000	718.00397	2.00000	**0.235242457900804**
ABC	70.00000	90.00000	1.00000	317.17055	2.00000	**0.235242457900804**
KH	70.00000	90.00000	1.00000	481.07988	2.00000	0.235242458886112
WOA	70.00000	90.00000	1.00000	182.35543	2.00000	0.235242457901052
HHO	70.00000	90.00000	1.00000	304.20738	2.00000	**0.235242457900804**
BOA	67.72699	90.00000	1.00000	673.06921	2.00000	0.248171278270212
HGSO	69.99945	90.00000	1.00000	8.73600	2.00000	0.235248138956563
LIACOR	70.00000	90.00000	1.00000	169.99845	2.00000	**0.235242457900804**
SMO	70.000	90.000	1.000	999.99	2.000	**0.2352424579**
ỌMOA	80.000	90.005	1.000	1000.0	2.000	**0.1250434142**

ỌMOA performed better than ABC, which was reported as best at the time of competition report, and others in this design problem and further set a much better global optimum than benchmarked in [42]; 0.23524245790; with the other algorithms as presented in [49].

IV. CONCLUSION

In this work, Ọdịgbo Metaheuristic Optimization Algorithm – ỌMOA, a new nature-inspired population-based metaphor, was proposed and used in experiments and engineering designs with very great performance. The idea stemmed from the informal learning pattern and discipleship, which is ingrained in the socio-cultural behavior of the indigenous peoples - the Ndigbo of a West African tribe is presented. The learners cope through practice and observation. The experiment conducted considered 30 benchmark unconstraint problems, 29 CEC 2017 (50D) real-parameter single objective constraint optimization, and about 8 engineering design constrained problems from CEC 2020; the results showed that OMOA had balanced exploitation and exploratory capacities with very good convergence time too. Comparison to the performance of other well-established state-of-the-art algorithms depicts the exceptional performance of the automata. The significant test also confirms the relative efficiency of ỌMOA with t-values and p-values presented in Table VIII and summarized in Table IX. The convergence time test using the Rastrigin function also shows OMOA had better speed than the contender in Table X. The competing algorithms were the most award winners in past competitions from 2017 till date. In all complex engineering problems presented, ỌMOA had performed remarkably well and had, in some cases, set new minimum attainable best solutions; Of interest are the new values better than the set global optimums in some functions and engineering designs (Clutch Disc, Himmelblau, GTCD, Rolling Bearing, Robotic Gripper).

The future direction is to further validate with the most recent CECs and design optimization problems in other fields. Meanwhile, ỌMOA shows merit to be considered in the current state-of-the-art.

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