Iterative Learning Control for High Relative Degree Discrete-Time Systems with Random Initial Shifts

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***Abstract*—In this paper, an iterative learning control (ILC) strategy under compression mapping framework is presented for high relative degree discrete-time systems with random initial shifts. Firstly, utilizing the high relative degree of the system and difference term, a control law is designed and a p-order non-homogeneous linear difference equation is established. The appropriate control gain is selected according to the characteristics of solution of the difference equation and the initial shifts, so as to ensure that the high relative degree discrete-time system can reach a steady-state deviation output at a fixed time. Subsequently, a PD-type control law is employed to correct the fixed deviation of the system. Theoretical analysis indicates that this ILC strategy can ensure that the high relative degree systems achieve accurate tracking after the predefined time. Finally, the simulation experiments are conducted on a linear discrete-time Multiple-Input Multiple-Output(MIMO) system with relative degree 1 and a Multiple-Input Single-Output(MISO) system with relative degree 2, respectively, and the results verify the effectiveness of the algorithm.**

***Keywords*—Relative degree; iterative learning control; random initial shifts; difference equation; discrete-time system**

I. Introduction

For the control systems in the field of repetitive operations, iterative learning control (ILC) is a commonly used intelligent control strategy. Drawing from prior batch tracking errors and inputs to update the current batch's control inputs, ILC enables the repetitively operated system to follow the desired trajectory to a high degree of precision over a finite interval. Notably, ILC's lack of reliance on the knowledge of system dynamics, coupled with its superior adaptability, renders it an ideal fit for complex control systems. ILC is widely used in robot control systems [1]-[3], medical rehabilitation [4], [5], multi-agent formation [6], [7], batch processes [8], [9], train automatic control [10], [11]and so on.

The relative degree is utilized to express the extent to which a system control input directly feeds back the system output. In mathematical terms, the relative degree is defined as the lowest order derivative of system output with respect to time, which can be directly fed back by the control inputs. In the discrete-time dynamic systems, the relative degree is manifested as the time delay between the input and output of system, which is inherent in many practical applications. In many engineering practices, the system relative degree is larger than 1. The widespread presence of high relative degree dynamic systems has incited considerable interest in their ILC research within the control community in recent years.

For the high relative degree nonlinear continuous systems, under strict conditions of zero initial error, [12] adopted an antagonistic ILC method to enable system convergence; [13] designed a class of ILC algorithms based on data sampling; [14] proposed ILC laws which using error derivatives with the order less than the system relative degree. [15] presented a first-order D-type ILC based on the dummy model, which does not require the relative degree to be known. When there is a fixed initial shift, the control law proposed in [16] achieved consistent tracking over a specified interval by incorporating an initial correction behavior. This correction strategy had also been used in [17], [18] for high relative degree nonlinear discrete-time systems with fixed initial states.

For the high relative degree linear continuous systems, when the initial error is 0, [19] proposed a linear matrix inequality(LMI) design method based on the bounded real lemma(BRL). The research in [20] presented a unified 2-D analysis method for both continuous and discrete-time systems by defining similar symbols for continuous and discrete operators, but the model can only achieve asymptotic tracking for systems with initial shifts. The study in [21] proposed a PD-type control law for the fixed initial shifts, which guarantees convergence of the system within a finite interval, and the convergence speed is uniform. For uncertain systems with fixed initial shifts, [22] proposed an adaptive ILC algorithm. For the high relative degree linear multi-variable discrete-time systems, [23] proposed an iterative learning controller with an H_∞-based approach to suppress the random iteration-varying perturbations; when the system has a fixed iteration initial error, a P-type ILC algorithm is presented in [24] can achieve asymptotic tracking.

Regarding the initial value problem of ILC, most of the studies require that the initial shift is zero or fixed [25]-[27]. However, in many practical situations, the system will always inevitably exist initial shifts at each iteration, and due to the limitation of the actual repeat localization accuracy, the study of ILC with arbitrary initial shifts is of great significance. For nonlinear systems with varying initial iteration errors and tracking trajectories, the study in [28] proposed two adaptive ILC laws to achieve a complete reference trajectory tracking. The study in [29] presented a ILC method with a time-varying sliding mode, which enables random initial state errors to converge to zero beyond a initial time interval. This strategy achieves complete tracking for second-order nonlinear systems. An adaptive ILC algorithm based on filtering error correction is proposed in [30] to achieve precise tracking for non-parametric uncertain systems with random initial shifts and unknown input dead zones. For linear discrete time-delay systems, [31] proposed an ILC strategy with correction of initial state deviation to solve the trajectory tracking problem. The research in [32] adopted a phased ILC strategy, which first corrected arbitrary initial deviation to fixed deviation, and then corrected the fixed deviation, to achieve complete tracking for second-order continuous systems.

Despite these remarkable advances on ILC for arbitrary initial value problems, the study of high relative degree systems with arbitrary initial shifts is still relatively scarce. For the linear continuous MIMO systems with vector relative degree, the study in [33] proposed a control strategy based on an iteratively moving average operator, which make the system converge under a fixed initial error condition. Under an arbitrary initial error condition, this algorithm only made the system converge to a bounded range. For high relative degree SISO continuous systems, the presented ILC algorithm in [34] is based on the high relative degree, which utilized a form of multi-pulse compensation to suppress arbitrary initial shifts. For high relative degree linear discrete-time MIMO systems, [35] presented three ILC algorithms based on average operator to achieve complete tracking under the condition that the initial state vibrate slightly near a fixed point.

In this paper, an ILC algorithm based on compression mapping are presented to solve the random initial shift problem for high relative degree linear discrete-time systems. This algorithm draws on the phased correction strategy in [32] to deal with the random initial deviation problem. The random initial shifts are transformed into a fixed shift using a difference controller, and then the fixed shift is corrected by a PD-type controller to make the system converge at a predefined time. Finally, the validity of the proposed algorithm is demonstrated by simulation of two examples with different relative degrees. The conclusions are presented.

II. Problem Formulation

Consider a linear discrete-time system operating in the interval $[0,T]$:

$$\begin{aligned} x_k(t+1) &= Ax_k(t)+Bu_k(t) \\ y_k(t) &= Cx_k(t) \end{aligned} \tag{1}$$

where, $k=1,2,\cdots$ denotes the number of iterations; $x_k(t)\in\mathbb{R}^n$, $u_k(t)\in\mathbb{R}^n$, $y_k(t)\in\mathbb{R}^n$ denote the state variable, input and output of the system, respectively. A,B,C are system parameter matrices, which B is right invertible and C is left invertible.

$y_d(t)$ is the given desired output, $x_d(t)$ is the corresponding desired state. The system output error is defined as follows:

$$e_k(t)=y_d(t)-y_k(t) \tag{2}$$

Definition 1: [35] For the linear discrete-time system (1), if the Markov parameters satisfy

$$\begin{cases} CA^iB=0, & 0\le i\le p-2 \\ CA^{p-1}B\neq 0. \end{cases} \tag{3}$$

The system relative degree of system is p.

Assumption 1: The initial state $x_k(0)\neq x_d(0)$ varies only arbitrarily within a certain range, i.e., $x_k(0)$ is a neighborhood of $x_d(0)$.

$$D=\{x_k(0)|\mid x_k(0)-x_d(0)\mid\le\frac{\Lambda}{2},x_k(0)\neq x_d(0)\}$$

where, $\frac{\Lambda}{2}$ is the radius of the neighborhood D.

III. Controller Design

In order to correct the random initial shifts of system (1), the controller is designed as follows:

$$\begin{aligned} & u_{k+1}(t) \\ = \; & u_k(t)+\textstyle\sum_{i=0}^{p-1}K_i(e_{k+1}(t-i)-e_k(t-i))+r_k(t) \end{aligned} \tag{4}$$

where, K_i are control gains that can be set manually. $r_k(t)$ is a undetermined function.

Considering $x_{k+1}(t+p)-x_k(t+p)$, there is

$$\begin{aligned} & x_{k+1}(t+p)-x_k(t+p) \\ = \; & A(x_{k+1}(t+p-1)-x_k(t+p-1)) \\ & +B(u_{k+1}(t+p-1)-u_k(t+p-1)) \end{aligned} \tag{5}$$

Combining the control law (4) and Eq. (5), there is

$$\begin{aligned} & x_{k+1}(t+p)-x_k(t+p) \\ = \; & A(x_{k+1}(t+p-1)-x_k(t+p-1)) \\ & +B\textstyle\sum_{i=0}^{p-1}K_i(e_{k+1}(t+p-1-i)-e_k(t+p-1-i)) \\ & +Br_k(t+p-1) \\ = \; & A(x_{k+1}(t+p-1)-x_k(t+p-1)) \\ & -B\textstyle\sum_{i=0}^{p-1}K_iC(x_{k+1}(t+p-1-i)-x_k(t+p-1-i)) \\ & +Br_k(t+p-1) \end{aligned}$$

Setting $\eta_k(t)=x_{k+1}(t)-x_k(t)$, there is

$$\begin{aligned} & \eta_k(t+p)+(BK_0C-A)\eta_k(t+p-1) \\ & +B\textstyle\sum_{i=1}^{p-1}K_iC\eta_k(t+p-1-i) \\ = \; & Br_k(t+p-1) \end{aligned} \tag{6}$$

Eq. (6) is a p-order linear non homogeneous difference equation with constant coefficients. The corresponding homogeneous difference equation is

$$\begin{aligned} & \eta_k(t+p)+(BK_0C-A)\eta_k(t+p-1) \\ & +B\textstyle\sum_{i=1}^{p-1}K_iC\eta_k(t+p-1-i)=0 \end{aligned} \tag{7}$$

Let its general solution be as follows:

$$\eta_k(t)=\varphi_k(t)+\varphi_k^*(t)$$

where $\varphi_k(t)$ is the general solution of Eq. (7) and $\varphi_k^*(t)$ is a particular solution of Eq. (6). One can set $\varphi_k(t)=\sum_{j=1}^{p}C_j\lambda_j^t$. λ_j are the p characteristic roots of the characteristic Eq. $\lambda^p+(BK_0C-A)\lambda^{p-1}+B\sum_{i=1}^{p-1}K_iC\lambda^{p-1-i}=0$; C_j are arbitrary constant matrices.

Let characteristic equation have only one root λ, there is

$$\lambda=\lambda_j=\frac{A-BK_0C}{p} \tag{8}$$

The control gains K_i $(i=1,2,\cdots,p-1)$ can be obtained as follows:

$$K_i=B^{-1}C_p^{i+1}(-\lambda)^{i+1}C^{-1}=\frac{B^{-1}p!(-\lambda)^{i+1}C^{-1}}{(p-i-1)!(i+1)!} \tag{9}$$

Then $\varphi_k(t)$ can be written as:

$$\varphi_k(t)=\textstyle\sum_{s=0}^{p-1}C_st^s\lambda^t$$

For convenience, one can set $r_k(t+p-1)=Q\lambda^t$, where Q is an undetermined constant matrix. There is $r_k(t)=Q\lambda^{t-p+1}$. There is

$$\varphi_k^*(t)=qt^p\lambda^t$$

where q is an undetermined constant matrix.

Substituting the particular solution $\varphi_k^*(t)$ and Eq. (9) into Eq. (5), there is

$$\begin{array}{rl} & BQ\lambda^t \\ = & q(t+p)^p\lambda^{t+p} + (BK_0C - A)q(t+p-1)^p\lambda^{t+p-1} \\ & +B\sum_{i=1}^{p-1} K_iC(t+p-i-1)^p q\lambda^{t+p-i-1} \\ = & ((t+p)^p - p(t+p-1)^p \\ & +\sum_{i=1}^{p-1} C_p^{i+1}(-1)^{i+1}(t+p-i-1)^p)q\lambda^{t+p} \\ = & \sum_{j=0}^{p} C_p^j(-1)^j(t+p-j)^p q\lambda^{t+p} \\ = & \sum_{j=0}^{p} C_p^j(-1)^j(-j)^p q\lambda^{t+p} \end{array} \tag{10}$$

Thus

$$Q = B^{-1}\sum_{j=0}^{p} C_p^j(-1)^j(-j)^p q\lambda^p \tag{11}$$

$\eta_k(t)$ can be represented as

$$\begin{array}{rl} & \eta_k(t) = \sum_{s=0}^{p-1} C_s t^s\lambda^t + qt^p\lambda^t \\ = & \sum_{s=0}^{p} C_s t^s \lambda^t \end{array} \tag{12}$$

where $C_p = q$.

When $t \in [0, p-1]$, there are

$$\begin{cases} \eta_k(0) = C_0 \\ \eta_k(1) = \sum_{s=0}^{p} C_s\lambda \\ \eta_k(2) = \sum_{s=0}^{p} C_s 2^s\lambda^2 \\ \vdots \\ \eta_k(p-1) = \sum_{s=0}^{p} C_s(p-1)^s\lambda^{p-1} \end{cases} \tag{13}$$

If at a certain time $t = h$, there is $\eta_k(h) \to 0$, thus

$$\eta_k(h) = \sum_{s=0}^{p} C_s h^s \lambda^h = 0 \tag{14}$$

From (13) and (14), the coefficients C_s in (12) can be obtained as follows.

$$\begin{pmatrix} C_0 \\ C_1 \\ C_2 \\ \vdots \\ C_{p-1} \\ C_p \end{pmatrix} = \Upsilon \begin{pmatrix} \eta_k(0) \\ \eta_k(1) \\ \eta_k(2) \\ \vdots \\ \eta_k(p-1) \\ 0 \end{pmatrix} \tag{15}$$

where,

$$\Upsilon = \begin{pmatrix} I & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \lambda & \lambda & \lambda & \cdots & \lambda & \lambda \\ \lambda^2 & 2\lambda^2 & 2^2\lambda^2 & \cdots & 2^{p-1}\lambda^2 & 2^p\lambda^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda^{p-2} & (p-2)\lambda^{p-2} & (p-2)^2\lambda^{p-2} & \cdots & (p-2)^{p-1}\lambda^{p-2} & (p-2)^p\lambda^{p-2} \\ \lambda^{p-1} & (p-1)\lambda^{p-1} & (p-1)^2\lambda^{p-1} & \cdots & (p-1)^{p-1}\lambda^{p-1} & (p-1)^p\lambda^{p-1} \\ \lambda^h & h\lambda^h & h^2\lambda^h & \cdots & h^{p-1}\lambda^h & h^p\lambda^h \end{pmatrix}^{-1}$$

When $t = h\ (h > p)$ and $\eta_k(h) \to 0$, there are $x_{k+1}(h) \to x_k(h)$, $x_k(h) \to x_{k-1}(h)$, $\cdots$, $x_3(h) \to x_2(h)$,. That is, when $t = h$, the system output $y_{k+1}(h)$ tends to a certain steady value, but not necessarily $y_d(h)$.

In order to converge the output error to zero, the controller is modified as follows:

$$u_{k+1}(t) = \begin{cases} u_k(t) + \sum_{i=0}^{p-1} K_i(e_{k+1}(t-i) - e_k(t-i)) & \\ +r_k(t) & t \in [0, h] \\ u_k(t) + K_d(e_{k+1}(t) - e_k(t)) & \\ +\Gamma e_k(t+p) & t \in (h, T] \end{cases} \tag{16}$$

where K_d and Γ are control gains that can be set manually.

IV. Convergence Analysis

This section focuses on he convergence analysis of the system (1) after applying the control law (16). In this section, $||\cdot||$ is defined to be a certain norm for vectors or matrices.

Theorem 1: When the initial shifts satisfies **Assumption 1**, and the control gain Γ satisfies

$$||I - B\Gamma CA^{p-1}|| < 1 \tag{17}$$

Then the correction control law (16) can make the system (1) achieve complete tracking,that is $\lim_{k\to\infty} ||e_{k+1}(t)|| = 0$.

Proof: According to the **Definition 1**, when $t \in [0, h]$, $CA^{i-1}B = 0$, one has

$$\begin{array}{rl} & y_k(t+i) \\ = & Cx_k(t+i) \\ = & CAx_k(t+i-1) + CBu_k(t+i-1) \\ = & CA^2x_k(t+i-2) + CABu_k(t+i-2) \\ & +CBu_k(t+i-1) \\ = & CA^ix_k(t) + \sum_{j=1}^{i} CA^{j-1}Bu_k(t+i-j) \\ = & CA^ix_k(t) \end{array} \tag{18}$$

In the analysis of the previous section, through the correction of the control law (16), the output error of the system (1) is stabilized at a fixed value when $t = h$.

When $t > h$, one has

$$\begin{array}{rl} & x_{k+1}(t) - x_k(t) \\ = & A(x_{k+1}(t-1) - x_k(t-1)) \\ & +BK_d(e_{k+1}(t-1) - e_k(t-1)) \\ & +B\Gamma e_k(t+p-1) \end{array} \tag{19}$$

From the Eq. (18), there is

$$e_k(t+p-1) = CA^{p-1}(x_d(t) - x_k(t)) \tag{20}$$

Setting $\Delta x_k(t) = x_d(t) - x_k(t)$. Substituting $\Delta x_k(t)$ into (19), one has

$$\begin{array}{rl} & \Delta x_k(t) - \Delta x_{k+1}(t) \\ = & (A - BK_dC)(\Delta x_k(t-1) - \Delta x_{k+1}(t-1)) \\ & +B\Gamma CA^{p-1}\Delta x_k(t) \end{array} \tag{21}$$

When $t = h+1$, according to $x_k(h) = x_{k+1}(h)$ and (21), there is

$$\Delta x_{k+1}(h+1) = (I - B\Gamma CA^{p-1})\Delta x_k(h+1)$$

Therefore, when $||I - B\Gamma CA^{p-1}|| < 1$, there is

$$\lim_{k\to\infty} ||\Delta x_{k+1}(h+1)|| = 0 \tag{22}$$

So

$$\begin{aligned} & \lim_{k\to\infty} ||e_{k+1}(h+1)|| \\ = & \; C \lim_{k\to\infty} ||\Delta x_{k+1}(h+1)|| \\ = & \; 0 \end{aligned}$$

Similarly, when $t \in (h+1, T]$, there is

$$\lim_{k\to\infty} ||e_{k+1}(t)|| = 0$$

V. Numerical Simulations

To verify the above conclusion, this paper conducts simulation experiments on two systems with relative degree 1 and 2, respectively. And compared it with the algorithm proposed in [24].

A. Linear Discrete-time MIMO System with Relative Degree 1

Considering the following MIMO system:

$$\begin{aligned} x_k(t+1) &= \begin{bmatrix} 0.6 & 0.25 \\ 0 & 0.65 \end{bmatrix} x_k(t) + \begin{bmatrix} 1 & 0.05 \\ 0.1 & 1.7 \end{bmatrix} u_k(t) \\ y_k(t) &= \begin{bmatrix} 0.8 & -0.13 \\ 0.1 & 0.5 \end{bmatrix} x_k(t) \end{aligned} \tag{23}$$

From $CB \neq 0$, the system relative degree is $p = 1$. Let the control gains of (16) be

$$K_0 = 1.2, \qquad K_d = 0.9, \qquad \Gamma = \begin{bmatrix} 0.27 & 0.14 \\ 0.03 & 0.31 \end{bmatrix}$$

It is easy to verify that $||I - B\Gamma CA^{p-1}|| < 1$, which satisfies the condition of **Definition 1**. $r_k(t)$ is determined according to Eq. (11) and (15). The system reference trajectory is as follows:

$$y_d(t) = \begin{bmatrix} y_{1,d}(t) \\ y_{2,d}(t) \end{bmatrix} = \begin{bmatrix} 0.0008(t-50)^2 - 1 \\ sin(0.02\pi t) \end{bmatrix} \tag{24}$$

The system initial state is $x_k(0) = [rand + 0.5 \;\; rand - 0.5]^T$(where $rand$ generates a random value between 0 and 1). Let the system operating interval be $[0, 100]$, and the preset time $h = 15$. The simulation results are shown in Fig. $1-3$, where simulation is implemented for 50 iterations.

Fig. 1 shows the results of system outputs $y_{1,k}(t)$ and $y_{2,k}(t)$ tracking the reference trajectories $y_{1,d}(t)$ and $y_{2,d}(t)$ after different number of iterations, respectively. Fig. 2 shows the results of tracking errors $e_{1,k}(t)$ and $e_{2,k}(t)$. Fig. 3 shows the variations of system inputs $u_{1,k}(t)$ and $u_{2,k}(t)$. From Fig. $1-3$, it is obvious that when $t = 15$, the system outputs $y_{1,k}(t)$ and $y_{2,k}(t)$ tend to be fixed values, and when $t = 16$, both the system tracking errors are zero, the system achieves accurate tracking.

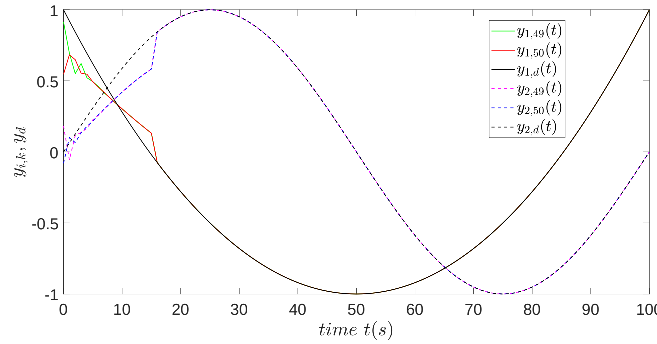

Fig. 1. Outputs $y_{i,k}(t)$ and reference trajectories $y_{i,d}(t)$.

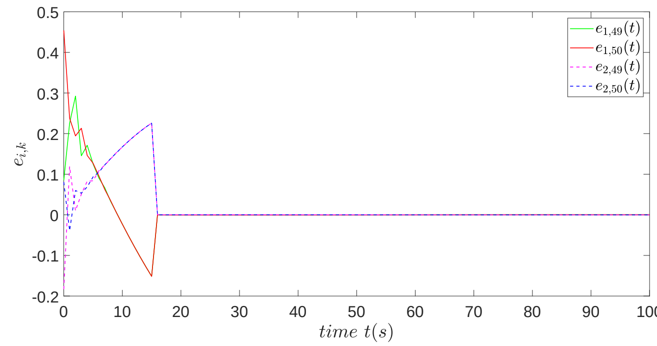

Fig. 2. Errors $e_{1,k}(t)e_{2,k}(t)$.

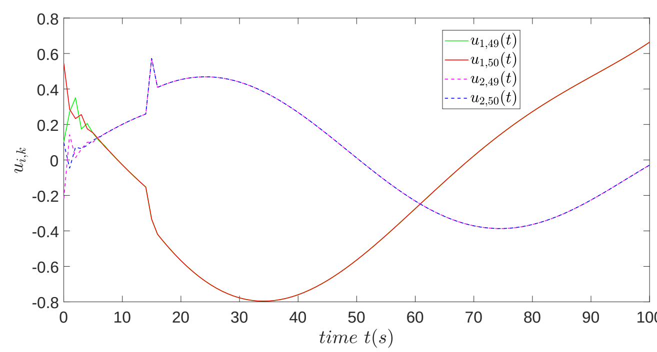

Fig. 3. Inputs $u_{1,k}(t)u_{2,k}(t)$.

B. Linear Discrete-time MISO System with Relative Degree 2

Considering the following MISO system:

$$\begin{cases} x_k(t+1) = \begin{bmatrix} 0.8 & 0.1 \\ -0.25 & -0.33 \end{bmatrix} x_k(t) + \begin{bmatrix} 0 \\ 1.1 \end{bmatrix} u_k(t) \\ y_k(t) = \begin{bmatrix} 1.7 & 0 \end{bmatrix} x_k(t) \end{cases} \tag{25}$$

From $CB = 0$ and $CAB \neq 0$, the system relative degree is $p = 2$. Let the control gains in the control law (16) be $K_0 = 1.8, K_d = 1.1, \Gamma = 2.9$. According to the relative degree $p = 2$ and Eq. (9), there exists $K_1 = -0.0052$. $r_k(t)$ is determined according to Eq. (11) and (15). It is also easy to verify that $||I - B\Gamma CA^{p-1}|| < 1$. Let the system operating interval be $[0, 100]$, and the preset time $h = 20$. The system desired trajectory is as follows:

$$y_d(t) = cos(0.02\pi t) \qquad t \in [1, 100]$$

The system initial state is $x_k(0) = [rand \quad rand]^T$. The simulation results are shown in Fig. $4-6$, where simulation is implemented for 50 iterations. Fig. 4 shows the result of the system output $y_k(t)$ tracking the desired trajectory $y_d(t)$ with the number of iterations. Fig. $5-6$ shows the variations of tracking error $e_k(t)$ and system input $u_k(t)$ after different number of iterations, respectively.

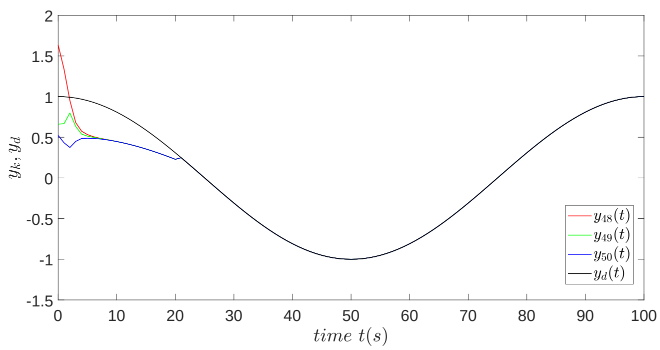

Fig. 4. $y_k(t)$ and $y_d(t)$.

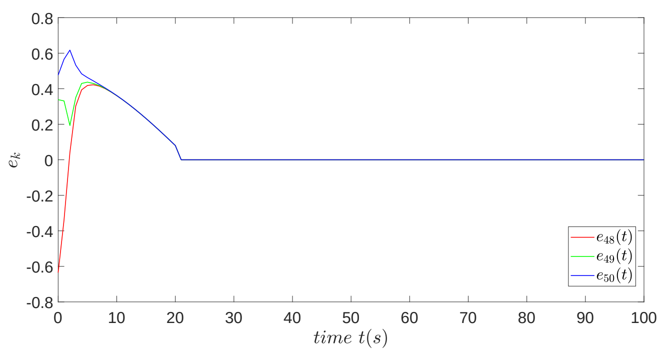

Fig. 5. Errors $e_k(t)$.

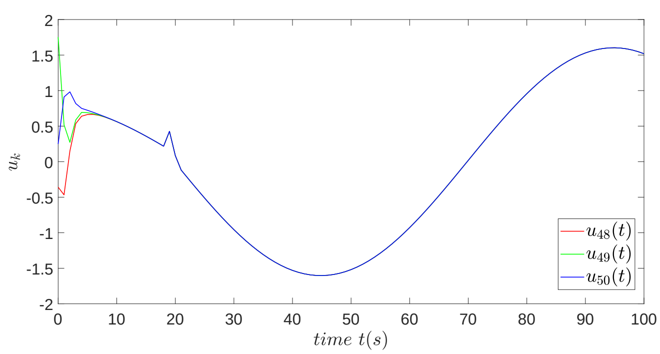

Fig. 6. Inputs $u_k(t)$.

From Fig. $4-6$, it is obvious that when $t = 20$, the system output tends to a fixed value. When $t = 21$, the system tracking error is 0, and the system achieves accurate tracking.

C. Comparison of Different Algorithms

For high relative degree linear discrete-time systems, the current research mainly focuses on the systems with fixed initial shifts, which is not applicable to the ones with random initial shifts. The ILC law (16) is compared with the algorithm proposed in [24] to demonstrate the effectiveness of the algorithm presented in this paper. Under the same conditions, the system (25) is simulated by using these two algorithms separately, and the results are shown in Fig. 7. The blue lines denote the outputs of the algorithm proposed in [24] after 48, 49, and 50 iterations respectively, and the black solid line denotes the desired trajectory, the red lines denote the outputs of the algorithm proposed in this paper. It is obvious that our algorithm can make the system converge, while the algorithm in [24] is unable to do.

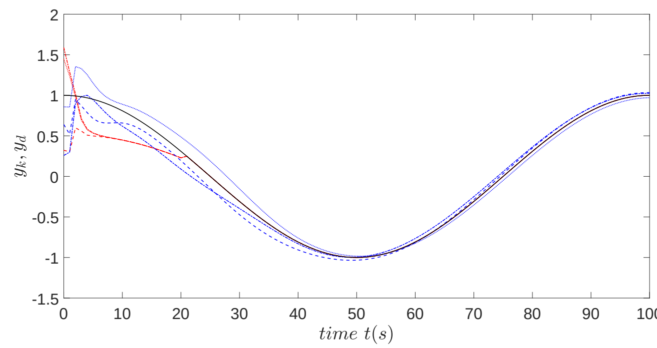

Fig. 7. $y_k(t)$ and $y_d(t)$.

VI. Conclusion

The ILC problem for high relative degree linear discrete-time systems with random initial shifts is discussed in this paper, and a control strategy with deviation correction is proposed. Theoretical analysis indicates that the presented algorithm can quickly correct the system initial state error and make the system converge after the predefined time. Finally, simulations were conducted using two discrete-time systems with relative degree 1 and 2, respectively, and the results proved the effectiveness of the algorithm. In the future, the effectiveness of this ILC strategy on discrete time-delay systems will be discussed.

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References

[1] C. E. Boudjedir, M. Bouri and D. Boukhetala, "Model-Free Iterative Learning Control With Nonrepetitive Trajectories for Second-Order MIMO Nonlinear Systems Application to a Delta Robot," IEEE Transactions on Industrial Electronics, vol. 68, no. 8, pp. 7433-7443, Aug. 2021.

[2] S. Chen, S. Hsieh and T. Ta, "Iterative learning contouring control for five-axis machine tools and industrial robots," Mechatronics, vol. 94, pp. 103-030, Oct. 2023.

[3] D. X. Ba, N. T. Thien and J. Bae, "A Novel Iterative Second-Order Neural-Network Learning Control Approach for Robotic Manipulators," IEEE Access, vol. 11, pp. 58318-58332, 2023.

[4] V. Molazadeh, Q. Zhang, X. Bao and N. Sharma, “An Iterative Learning Controller for a Switched Cooperative Allocation Strategy During Sit-to-Stand Tasks with a Hybrid Exoskeleton,” IEEE Transactions on Control Systems Technology, vol. 30, no. 3, pp. 1021-1036, May 2022.

[5] L. Liu, M. Illian, S. Leonhardt and B. J. E. Misgeld, “Iterative Learning Control for Cascaded Impedance-Controlled Compliant Exoskeleton With Adaptive Reaction to Spasticity,” IEEE Transactions on Instrumentation and Measurement, vol. 72, pp. 1-11, 2023.

[6] R. Hou, L. Cui, X. Bu and J. Yang, “Distributed formation control for multiple non-holonomic wheeled mobile robots with velocity constraint by using improved data-driven iterative learning,” Applied Mathematics and Computation, vol. 395, pp. 125-829, Apr. 2021.

[7] X. Fu and J. Peng, “Iterative learning control for UAVs formation based on point-to-point trajectory update tracking,” Mathematics and Computers in Simulation, vol. 209, pp. 1-15, July 2023.

[8] H. Shokri-Ghaleh, S. Ganjefar and A. M. Shahri, “Extension of iterative learning control design for batch processes with time-delay in the input subject to random cycle-varying uncertainties,” Journal of the Franklin Institute, vol. 360, no. 12, pp. 8528-8549, Dec. 2023.

[9] H. Li, S. Wang, H. Shi, C. Su and P. Li, “Two-Dimensional Iterative Learning Robust Asynchronous Switching Predictive Control for Multiphase Batch Processes With Time-Varying Delays,” IEEE Transactions on Systems, Man, and Cybernetics: Systems, vol. 53, no. 10, pp. 6488-6502, Oct. 2023.

[10] Y. Chen, D. Huang, Y. Li and X. Feng, “A Novel Iterative Learning Approach for Tracking Control of High-Speed Trains Subject to Unknown Time-Varying Delay,” IEEE Transactions on Automation Science and Engineering, vol. 19, no. 1, pp. 113-121, Jan. 2022.

[11] J. Zheng and Z. Hou, “Data-Driven Spatial Adaptive Terminal Iterative Learning Predictive Control for Automatic Stop Control of Subway Train With Actuator Saturation,” IEEE Transactions on Intelligent Transportation Systems, vol. 24, no. 10, pp. 11453-11465, Oct. 2022.

[12] M. Sun and D. Wang, “Anticipatory Iterative Learning Control for Nonlinear Systems with Arbitrary Relative Degree,” IEEE Transactions on automatic control, vol. 46, no. 5, pp. 783-788, 2001.

[13] M. Sun and D. Wang, “Sample-data iterative learning control for nonlinear systems with arbitrary relative degree,” Automatic, vol. 37, no. 2, pp. 283-289, 2001.

[14] M. Sun and D. Wang, “Higher relative degree nonlinear systems with ILC using lower-order differentiations,” Asian Journal of Control, vol. 4, no. 1, pp. 38-48, 2002.

[15] Z. Song, J. Mao and S. Dai, “First-order D-type Iterative Learning Control for Nonlinear Systems with Unknown Relative Degree,” Acta Automatica Sinica, vol. 31, no. 4, pp. 555-561, 2005.

[16] M. Sun and D. Wang, “Iterative learning control with initial rectifying action,” Automatic, vol. 38, no. 7, pp. 1177-1182, 2002.

[17] M. Sun and D. Wang, “Analysis of nonlinear discrete-time systems with higher-order iterative learning control,” Dynamics and Control, vol. 11, no. 1, pp. 81-96, 2001.

[18] M. Sun and D. Wang, “Initial shift issues on discrete-time iterative learning control with system relative degree,” IEEE Trans. Autom. Control, vol. 48, no. 1, pp. 144-148, 2003.

[19] D. Meng, Y. Jia, Du J and F. Yu, “Monotonically convergent ILC systems designed using bounded real lemma,” International Journal of Systems Science, vol. 43, no. 11, pp. 2062-2071, 2012.

[20] D. Meng, Y. Jia, Du J and F. Yu, “Data-driven control for relative degree systems via iterative learning,” IEEE Transactions on Neural Networks, vol. 22, no. 12, pp. 2213-2225, 2011.

[21] Q. Fu, L. Du, G. Xu, J. Wu and P. Yu, “PD-type iterative learning control for linear continuous systems with arbitrary relative degree,” Transactions of the Institute of Measurement and Control, vol. 41, no. 9, pp. 2555-2562, 2019.

[22] C. Chien and C. Yao, “An output-based adaptive iterative learning controller for high relative degree uncertain linear systems,” Automatic, vol. 40, no. 1, pp. 145-153, 2004.

[23] D. Meng, Y. Jia, Du J and F. Yu, “Robust learning controller design for MIMO stochastic discrete-time systems: An H_∞-based approach,” International Journal of Adaptive Control and Signal Processing, vol. 25, pp. 653-670, 2011.

[24] Z. Mao and X. Li, “Iterative learning control for linear discrete systems with high relative degree and iterative initial error,” Control Theory & Applications, vol. 29, no. 8, pp. 1078-1081, 2012.

[25] D. Huang, J. Xu, X. Li X, et al, “D-type anticipator iterative learning control for a class in homogeneous heat equations,” Automatica, vol. 49, pp. 2397-2408, 2013.

[26] X. Dai, C. Xu, S. Tian and Z. Li, “Iterative learning control for MIMO second-order hyperbolic distributed parameter systems with uncertainties,” Advances in Difference Equations, vol. 94, pp. 1-13, 2016.

[27] P. GU, Q. FU and J. WU, “State Tracking Algorithm for Linear Singular Iterative Learning Control Systems with Fixed Initial Shift,” Mathematica Applicata, vol. 30, no. 1, pp. 8-15, 2017.

[28] X. Li, M. Lv and K. John, “Adaptive ILC algorithms of nonlinear continuous systems with non-parametric uncertainties for non-repetitive trajectory tracking,” International Journal of Systems Science, vol. 47, no. 10, pp. 2279-2289, 2016.

[29] C. Yin, S. Riaz, H. Zaman, N. Ullah, V. Blazek, L. Prokop and S. Misak, “A Novel Predefined Time PD-Type ILC Paradigm for Nonlinear Systems,” mathematics, vol. 11, no. 1, 2023.

[30] Q. Yan, M. Sun and J. Cai, “Filtering-error rectified iterative learning control for systems with input dead-zone,” Control Theory Applications, vol. 34, no. 1, pp. 77-84, 2017.

[31] G. Li, D. Chen, Q. Dong and K. Wang, “Iterative learning control for a class of discrete-time systems with time delay and random initial state errors,” Pure and Applied Mathematics, vol. 38, no. 2, pp. 224-235, 2022.

[32] D. Chen, Y. Xu, T. Lu and G. Li, “Multi-phase iterative learning control for high-order systems with arbitrary initial shifts,” Mathematics and Computers in Simulation, vol. 216, pp. 231-245, 2024.

[33] Y. Wei and X. Li, “Robust iterative learning control for linear continuous systems with vector relative degree under varying input trail lengths and random initial state shifts,” International Journal of Robust and Nonlinear Control, vol. 31, no. 2, pp. 609-622, 2021.

[34] X. Ruan and J. Wang, “Iterative Learning Control for Linear Time-invariant Systems with Higher-order Relative Degree,” Acta Mathematicae Applicatae Sinica, vol. 37, no. 6, pp. 1077-1092, 2014.

[35] Y. Wei and X. Li, “Iterative learning control for linear discrete-time systems with high relative degree under initial state vibration,” IET Control Theory and Applications, vol. 10, no. 10, pp. 1115-1126, 2016.