A Hybrid Method to Improve Forecasting Accuracy in the Case of Sanitary Materials Data

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*Abstract***—Sales forecasting is a starting point of supply chain management, and its accuracy influences business management significantly. In industries, how to improve forecasting accuracy such as sales, shipping is an important issue. In this paper, a hybrid method is introduced and plural methods are compared. Focusing that the equation of exponential smoothing method (ESM) is equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in exponential smoothing method is proposed before by Takeyasu et.al. which satisfies minimum variance of forecasting error. Firstly, we make estimation of ARMA model parameter and then estimate smoothing constants. In this paper, combining the trend removing method with this method, we aim to improve forecasting accuracy. Trend removing by the combination of linear and 2nd order non-linear function and 3rd order nonlinear function is carried out to the manufacturer's data of sanitary materials.**

The new method shows that it is useful for the time series that has various trend characteristics and has rather strong seasonal trend. The effectiveness of this method should be examined in various cases.

Keywords—component; minimum variance; exponential smoothing method; forecasting; trend; sanitary materials

I. INTRODUCTION

The needs for sales forecasting is prevailing among companies, but the contents of such needs are undergoing significant changes because of the rapid changes in the recent business environment. Correct forecasting along with supply chain management is required that leads to the shortened lead time and less stocks.

Time series analysis is often used in such themes as sales forecasting, stock market price forecasting etc. Sales forecasting is inevitable for Supply Chain Management. But in fact, it is not well utilized in industries. It is because there are so many irregular incidents therefore it becomes hard to make sales forecasting. A mere application of method does not bear good result. The big reason is that sales data or production data are not stationary time series, while linear model requires the time series as a stationary one. In order to improve forecasting accuracy, we have devised trend removal methods as well as searching optimal parameters and obtained good results. We created a new method and applied it to various time series and examined the effectiveness of the method. Applied data are sales data, production data, shipping data, stock market price data, flight passenger data etc.

Many methods for time series analysis have been presented such as Autoregressive model (AR Model), Autoregressive Moving Average Model (ARMA Model) and Exponential Smoothing Method $(ESM)^{[1]-[4]}$. Among these, ESM is said to be a practical simple method.

For this method, various improving method such as adding compensating item for time lag, coping with the time series with trend $[5]$, utilizing Kalman Filter $[6]$, Bayes Forecasting^[7], adaptive ESM^[8], exponentially weighted

Moving Averages with irregular updating periods $[9]$, making averages of forecasts using plural method [10] are presented. For example, Maeda^[6] calculated smoothing constant in relationship with S/N ratio under the assumption that the observation noise was added to the system. But he had to calculate under supposed noise because he could not grasp observation noise.

It can be said that it does not pursue optimum solution from the very data themselves which should be derived by those estimation. Ishii $[11]$ pointed out that the optimal smoothing constant was the solution of infinite order equation, but he didn't show analytical solution. Based on these facts, a new method of estimation of smoothing constant in ESM was proposed before $[12]$. Focusing that the equation of ESM is equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in ESM was derived. Furthermore, combining the trend removal method, forecasting accuracy was improved, where shipping data, stock market price data etc. were examined $[13]$ -[19].

In this paper, utilizing above stated method, a revised forecasting method is proposed. A mere application of ESM does not make good forecasting accuracy for the time series which has non-linear trend and/or trend by month. A new method to cope with this issue is required. Therefore, utilizing above stated method, a revised forecasting method is proposed in this paper to improve forecasting accuracy. In making forecast such as production data, trend removing method is devised. Trend removing by the combination of linear and 2*nd* order non-linear function and 3*rd* order non-linear function is executed to the manufacturer's data of sanitary materials. The weights for these functions are set 0.5 for two patterns at first and then varied by 0.01 increment for three patterns and optimal weights are searched. For the comparison, monthly trend is removed after that. Theoretical solution of smoothing constant of ESM is calculated for both of the monthly trend removing data and the non-monthly trend removing data. Then forecasting is executed on these data. This is a revised forecasting method. Variance of forecasting error of this newly proposed method is assumed to be less than those of previously proposed method. The new method shows that it is useful especially for the time series that has stable characteristics and has rather strong seasonal trend and also the case that has non-linear trend. The rest of the paper is organized as follows. In section 2, the new method is described. ESM is stated by ARMA model and estimation method of smoothing constant is derived using ARMA model identification. The combination of linear and non-linear function is introduced for trend removing and the Monthly Ratio is also referred. Forecasting is executed in section 3, and estimation accuracy is examined, which is followed by the Discussion of section 4

II. DESCRIPTION OF THE NEW METHOD

A. Description of ESM Using ARMA Model[12]

In ESM, forecasting at time $t+1$ is stated in the following equation.

$$
\hat{x}_{t+1} = \hat{x}_t + \alpha (x_t - \hat{x}_t)
$$

= $\alpha x_t + (1 - \alpha) \hat{x}_t$ (1)

Here,

 \hat{x}_{t+1} : forecasting at $t+1$

 x_t : realized value at *t*

- α : smoothing constant $(0 < \alpha < 1)$
- (1) is re-stated as

$$
\hat{x}_{t+1} = \sum_{l=0}^{\infty} \alpha (1 - \alpha)^l x_{t-l}
$$
 (2)

By the way, we consider the following (1,1) order ARMA model.

$$
x_{t} - x_{t-1} = e_{t} - \beta e_{t-1}
$$
 (3)

Generally, (p,q) order ARMA model is stated as

$$
x_{t} + \sum_{i=1}^{p} a_{i} x_{t-i} = e_{t} + \sum_{j=1}^{q} b_{j} e_{t-j}
$$
 (4)

Here,

$$
\{x_{t}\}.
$$

Sample process of Stationary Ergodic Gaussian Process $x(t)$ $t = 1, 2, \dots, N, \dots$

$$
{ei}:\text{Gaussian White Noise with 0 mean }\sigma_{e}^{2}\text{ variance}
$$

MA process in (4) is supposed to satisfy convertibility condition. Utilizing the relation that

$$
E[e_t|e_{t-1}, e_{t-2}, \cdots] = 0
$$

we get the following equation from (3).

$$
\hat{x}_t = x_{t-1} - \beta e_{t-1} \tag{5}
$$

Operating this scheme on $t+1$, we finally get

$$
\hat{x}_{t+1} = \hat{x}_t + (1 - \beta)e_t \n= \hat{x}_t + (1 - \beta)(x_t - \hat{x}_t)
$$
\n(6)

If we set $1 - \beta = \alpha$, the above equation is the same with (1), i.e., equation of ESM is equivalent to (1,1) order ARMA model, or is said to be $(0,1,1)$ order ARIMA model because 1st order AR parameter is $-1^{[1][3]}$. Focusing that the equation of exponential smoothing method (ESM) is equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in exponential smoothing method is derived.

Finally we get:

$$
b_1 = \frac{1 - \sqrt{1 - 4\rho_1^2}}{2\rho_1}
$$

\n
$$
\alpha = \frac{1 + 2\rho_1 - \sqrt{1 - 4\rho_1^2}}{2\rho_1}
$$
\n(7)

Thus we can obtain a theoretical solution by a simple way.

Here P_1 must satisfy

$$
-\frac{1}{2} < \rho_1 < 0 \tag{8}
$$

in order to satisfy $0 < \alpha < 1$.

Focusing on the idea that the equation of ESM is equivalent to (1,1) order ARMA model equation, we can estimate smoothing constant after estimating ARMA model parameter.

It can be estimated only by calculating 0th and 1st order autocorrelation function.

B. Trend Removal Method[12]

As ESM is a one of a linear model, forecasting accuracy for the time series with non-linear trend is not necessarily good. How to remove trend for the time series with non-linear trend is a big issue in improving forecasting accuracy. In this paper, we devise to remove this non-linear trend by utilizing non-linear function.

As trend removal method, we describe the combination of linear and non-linear function.

[1] Linear function

We set

$$
y = a_1 x + b_1 \tag{9}
$$

as a linear function.

[2] Non-linear function

We set

$$
y = a_2 x^2 + b_2 x + c_2 \tag{10}
$$

$$
y = a_3 x^3 + b_3 x^2 + c_3 x + d_3 \tag{11}
$$

as a $2nd$ and a $3rd$ order non-linear function.

[3] The combination of linear and non-linear function We set

$$
y = \alpha_1 (a_1 x + b_1) + \alpha_2 (a_2 x^2 + b_2 x + c_2)
$$
 (12)

$$
y = \beta_1 (a_1 x + b_1) + \beta_2 (a_3 x^3 + b_3 x^2 + c_3 x + d_3)
$$
 (13)

$$
y = \gamma_1 (a_1 x + b_1) + \gamma_2 (a_2 x^2 + b_2 x + c_2)
$$

+ $\gamma_3 (a_3 x^3 + b_3 x^2 + c_3 x + d_3)$ (14)

as the combination of linear and $2nd$ order non-linear and 3^{rd} order non-linear function. Here, $\alpha_2 = 1 - \alpha_1$, $\beta_2 = 1 - \beta_1$, $\gamma_3 = 1 - (\gamma_1 + \gamma_2)$. Comparative discussion concerning (12), (13) and (14) are described in section 5.

C. Monthly Ratio[12]

For example, if there is the monthly data of L years as stated bellow:

$$
\left\{x_{ij}\right\}\left(i=1,\cdots,L\right)\left(j=1,\cdots,12\right)
$$

Where, $x_{ij} \in R$ in which *j* means month and *i* means year and x_{ij} is a shipping data of i-th year, j-th month. Then, monthly ratio \widetilde{X}_j $(j=1,\dots,12)$ is calculated as follows.

$$
\widetilde{x}_j = \frac{\frac{1}{L} \sum_{i=1}^{L} x_{ij}}{\frac{1}{L} \cdot \frac{1}{12} \sum_{i=1}^{L} \sum_{j=1}^{12} x_{ij}}
$$
(15)

Monthly trend is removed by dividing the data by (15). Numerical examples both of monthly trend removal case and non-removal case are discussed in 5.

III. FORECASTING THE SHIPPING DATA OF MANUFACTURER

A. Analysis Procedure

Manufacturer's data of sanitary materials from September 2009 to August 2012 are analyzed. First of all, graphical charts of these time series data are exhibited in Fig. 1, 2,3.

Fig. 1. Product A

Fig. 2. Product B

Fig. 3. Product C

Analysis procedure is as follows. There are 36 monthly data for each case. We use 24 data(1 to 24) and remove trend by the method stated in 2.2. Then we calculate monthly ratio by the method stated in 2.3. After removing monthly trend, the method stated in 2.1 is applied and Exponential Smoothing Constant with minimum variance of forecasting error is estimated.

Then 1 step forecast is executed. Thus, data is shifted to 2nd to 25th and the forecast for 26th data is executed consecutively, which finally reaches forecast of 36th data. To examine the accuracy of forecasting, variance of forecasting error is calculated for the data of 25th to 36th data. Final forecasting data is obtained by multiplying monthly ratio and trend. Forecasting error is expressed as:

$$
\varepsilon_i = \hat{x}_i - x_i \tag{16}
$$

$$
\bar{\varepsilon} = \frac{1}{N} \sum_{i=1}^{N} \varepsilon_i
$$
 (17)

B. Trend Removing

Trend is removed by dividing original data by $(12),(13),(14)$. The patterns of trend removal are exhibited in Table1.

In pattern1 and 2, the weight of α_1 , α_2 , β_1 , β_2 are set 0.5 in the equation (12),(13). In pattern3, the weight of α_1 is shifted by 0.01 increment in (12) which satisfy the range $0 \le \alpha_1 \le 1.00$. In pattern4, the weight of β_1 is shifted in the same way which satisfy the range $0 \le \beta_1 \le 1.00$. In pattern5, the weight of γ_1 and γ_2 are shifted by 0.01 increment in (14) which satisfy the range $0 \leq \gamma_1 \leq 1.00$, $0 \leq \gamma_2 \leq 1.00$. The best solution is selected which minimizes the variance of forecasting error. Estimation results of coefficient of (9), (10) and (11) are exhibited in Table 2. Estimation results of weights of (12), (13) and (14) are exhibited in Table 3.

Graphical chart of trend is exhibited in Fig. 4, 5, 6 for the cases that monthly ratio is used.

		1 Sl		γ nd		γ rd				
	a_{1}		a ₂	\mathcal{D}_2	c ₂	a ₃				
Pro duct A	43209.1095	5856643.29	1333.16432	9880.00155	6001069.43	567.0224981	19930.17936	226879.511 n	5503507.19	
Pro duct B	12078.8556	2983007.05	50.8956065	10806.4654	2988520.74	249.7081938	9414.952874	84756.8602	3207639.68	
Pro duct	12459.3434	3155302.37	397.796594	22404.2583	3112207.74 2	256.3640436	9215.855041	75706.2611	3337167.19	

TABLE I. COEFFICIENT OF (9),(10) AND (11)

TABLE II. THE PATTERNS OF TREND REMOVAL

Pattern1	α_1 , α_2 are set 0.5 in the equation (12)
Pattern ₂	β_1 , β_2 are set 0.5 in the equation (13)
Pattern ₃	α_1 is shifted by 0.01 increment in (12)
Pattern4	β_1 is shifted by 0.01 increment in (13)
Pattern ₅	γ_1 and γ_2 are shifted by 0.01 increment in (14)

Fig. 4. Trend of Product A

Fig. 5. Trend of Product B

Fig. 6. Trend of Product C

C. Removing trend of monthly ratio

After removing trend, monthly ratio is calculated by the method stated in 2.3.

Calculation result for 1st to 24th data is exhibited in Table 4 through 8.

D. Estimation of Smoothing Constant with Minimum Variance of Forecasting Error

After removing monthly trend, Smoothing Constant with minimum variance of forecasting error is estimated utilizing (7). There are cases that we cannot obtain a theoretical solution because they do not satisfy the condition of (A-9).

In those cases, Smoothing Constant with minimum variance of forecasting error is derived by shifting variable from 0.01 to 0.99 with 0.01 interval. Calculation result for 1st to 24th data is exhibited in Table 9.

E. Forecasting and Variance of Forecasting Error

Utilizing smoothing constant estimated in the previous section, forecasting is executed for the data of 25th to 36th data. Final forecasting data is obtained by multiplying monthly ratio and trend.

Variance of forecasting error is calculated by (18). Forecasting results are exhibited in Fig. 7, 8, 9 for the cases that monthly ratio is used.

TABLE V. MONTHLY RATIO (PATTERN2)

TABLE VII. MONTHLY RATIO (PATTERNA)

Month		∼								10	. .	\sim $\overline{1}$
Product A	1.10	1.01	0.92	1.19	.7 ² 0.14	0.91	0.99	0.89	1.03	1.07	\sim 1.14	1.05
Product B	0.97	1.16	0.83	1.02	.77 v. 1	0.84	15 1.1J	12° 1.JJ	0.80	1.00	1.05	1.09
Product C	1.06	1.06	1.00	1.23	0.84	0.68	1° 1.14	\sim π v. 1	0.03	1.06	1.20	0.96

TABLE VIII. MONTHLY RATIO (PATTERN5)

	Mont	Pattern1		Pattern ₂		Pattern3		Pattern4		Pattern5	
	hly ratio	\mathcal{D}_1	α		α		α	$\boldsymbol{\nu}$	α		α
Product	Used	-0.41	4 7 6 $\mathbf{0}$	-0.4423	0.3966	-0.397 6	.5050 θ	$\bf{0}$ - 0 42	4 5 3 0	-0 . 397 -6	.5050 $\overline{0}$
A	Not used	-0.2679	0.7095	-0.2864	0.6852	-0.2648	3 θ	$\mathbf Q$ - 0 6	80 $\overline{0}$	- 8 -0 64	35 Ω
Product B	Used	-0.2245	0.7628	-0.2357	7496 0 ¹	22 -0 46	.7 ' 6 2 8 θ	- 0 64	' 6 0 6	2 2 4 6 θ	7628 Ω
	Not used	-0.066	0.9337	-0.0749	0.9247	-0.0660	9 Ω	0 ₆ - 0 σ	9	0660 -0	9 3 3 7 $\bf{0}$
Product \mathcal{C}	Used	-0.4232	0.4476	-0.421	4 5 2 6 0 .	.4235 -0 .	0.4469	8 -0	4607	4 2 3 5 θ $\overline{}$.4469 Ω
	Not used	-0.266	0.7119	-0.2671	0.7106	-0.2656	25 θ	2 6 5 6 - 0		2 6 5 6 -0	25 71 Ω

TABLE IX. ESTIMATED SMOOTHING CONSTANT WITH MINIMUM VARIANCE

Fig. 7. Forecasting Results of Product A

Fig. 8. Forecasting Results of Product B

Fig. 9. Forecasting Results of Product C

Variance of forecasting error is exhibited in Table 10

	Monthly ratio	Pattern1	Pattern ₂	Pattern ₃	Pattern4	Pattern ₅	
Product	Used	841,383,645,691.7740	1,253,890,581,301.6900	828013457221.089	878641598931.531	828013457221.089	
A	Not used	1,233,162,627,302.1600	1,476,091,369,104.5700	1228777654815.98	1239890302140.87	1228777654815.98	
Product	Used	272,535,112,508.2970	298,632,461,408.8090	258441818256.66	276267182227.964	258441818256.66	
B	Not used	530, 376, 395, 688. 7940	578,434,563,503.2370	529971383044.765	539573173959.263	529971383044.765	
Product	Used	1,144,959,209,811.1200	1,361,652,912,074.6500	1144904100567.48	1161683354954.07	1144904100567.48	
C	Not used	2,104,397,068,344.4300	2,357,366,912,250.4600	2075183264170.54	2075183264170.54	2075183264170.54	

TABLE X. VARIANCE OF FORECASTING ERROR

F. Remarks

These time series have non-linear trend and trend by month. Applying only an ESM does not make good forecasting accuracy.

All cases had a good result in $1st+2nd$ order with the case that monthly ratio is used. We can observe that monthly trend is rather apparent in these cases. Therefore the method has selected the monthly trend removing case.

IV. DISCUSSION

Correct sales forecasting is inevitable in industries. Poor sales forecasting accuracy leads to a gap between the sales plan and result, which in turn generates a gap between the sales plan and the production plan. The condition in which the quantity in a production plan exceeds that in a sales plan (excess production) pushes up cost caused by increased finished and intermediate product inventory. Increased inventory and prolonged dwell time of product in inventory will lead to increased waste loss as well as extended lead-time, affecting customer satisfaction. In order to improve forecasting accuracy, we have devised trend removal methods as well as searching optimal parameters and obtained good results. We created a new method.

V. CONCLUSION

Focusing on the idea that the equation of exponential smoothing method(ESM) was equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in exponential smoothing method was proposed before by Takeyasu et.al.[12] which satisfied minimum variance of forecasting error. Combining the trend removal method with this method, we aimed to improve forecasting accuracy.

A mere application of ESM does not make good forecasting accuracy for the time series which has non-linear trend and/or trend by month. A new method to cope with this issue is required. Therefore, utilizing above stated method, a revised forecasting method is proposed in this paper to improve forecasting accuracy. An approach to this method was executed in the following method. Trend removal by a linear function was applied to the manufacturer's data of sanitary materials. The combination of linear and non-linear function was also introduced in trend removing. For the comparison, monthly trend was removed after that. Theoretical solution of smoothing constant of ESM was calculated for both of the monthly trend removing data and the non-monthly trend removing data.

Then forecasting was executed on these data. Product Ⅰ and Product II had a good result in $1st+2nd$ order with the case that monthly ratio is used, while Product Ⅲhad a good result in 1st+2nd order with the case that monthly ratio is not used.

VI. FUTURE WORKS

It is our future works to investigate much further cases to confirm the effectiveness of our new method. Various cases should be examined hereafter.

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