Fault-Tolerant Fusion Algorithm of Trajectory and Attitude of Spacecraft Based on Multi-Station Measurement Data

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***Abstract*—Aiming at the practical situation that the navigation processes of spacecrafts usually rely on several different kinds of tracking equipments which track the spacecraft by turns, a series of new outlier-tolerant fusion algorithms are build to determine the whole flight path as well as attitude parameters. In these new algorithms, the famous gradient descent methods are used to find out the outliers-tolerant flight paths from an integrated data-fusion function designed delicately. In this paper, these new algorithms are used to determine reliably the flight paths and attitude parameters in the situation that a spacecraft is tracked by a series of equipments working by turns and there are some outliers arising in the data series. Advantages of these new algorithms are not only plenary fusion of all of the data series from different kinds of equipments but also discriminatory usage: on the one hand, if the data are dependable, the useable information contained in these data are sufficiently used; on the other hand, if the data are outliers, the bad information from these data are efficiently eliminated from these algorithms. In this way, all of the computational flight paths and attitude parameters are insured to be consistent and reliable.**

***Keywords*—trajectory; fault-tolerance; data fusion**

I. Introduction

This It is necessary for a spacecraft that reliable TT&C network can track, measure and determine its trajectory during the whole flight process. TT&C network usually consists of optical measuring equipments (photoelectric theodolite and laser theadolite) and radio measuring equipments (pulse radar and continuous wave interferometer). Through these equipments partially overlapped relay tracking link, TT&C network can realize the tracking and measurement of spacecrafts in the expansive universe and the orientation and navigation during long-term operation [1-3].

Assuming that there are s_a laser theadolites, s_b pulse radars , s_c ground stations distributed s_d continuous wave interferometer system under the flight trajectory of a spacecraft, through the relay tracking of the partial overlapped link, that can obtain azimuth angle A, elevation angle E and radial distance R in the axis orthogonal coordinates of the spacecraft relative to each related ground station's equipments, as well as the radial distance difference P between two stations and the spacecraft. How to scientifically and effectively use the tracking data from different types of measurement equipments and accurately calculate the flight trajectory of the spacecraft is a research project with engineering background?

Currently, this problem is solved mainly by executing subsection calculation and piecewise series connection based on data rationality check. If subsection calculation result series connection method is used, it will unavoidably cause the lost of partial measurement data and sidesteps of several connection points, which make the trajectory calculation result incoherent. If the measurement data includes outliers, the conventional method even obtains a partial abnormal trajectory, which will influence the analysis on flight state of the spacecraft. This paper proposes and designs a rapid calculation method of spacecraft trajectory and attitude parameter based on fault-tolerance fusion during the whole tracking process.

II. Point-by-Point Fusion Calculation of Spacecraft Trajectory

In order to simply describe the algorithm, this paper divides the tracking data from $s_a + s_b + s_c + s_d$ measurement equipments on the tracking link of the spacecraft into 4 data types:

Type I: the radial distance data set $S_R = \{\{R_i(t), t \in [t_i^a, T_i^a]\}, i = 1,2,\cdots,s_1\}$, which means spacecraft- station ranging data of s_1 time periods with sampling interval of h s , the i th tracking interval is $[t_i^a, T_i^a]$, and the station's coordinate is $X_{ai} = (x_{ai}, y_{ai}, z_{ai})$;

Type II: the azimuth angle data set $S_A = \{\{A_i(t), t \in [t_i^b, T_i^b]\}, i = 1,2,\cdots,s_2\}$, which means the azimuth angle of the spacecraft relative to the station with sampling interval h s in s_2 time periods, the i th tracking interval is $[t_i^b, T_i^b]$, the coordinate of the station is $X_{bi} = (x_{bi}, y_{bi}, z_{bi})$ and the transformation matrix is Φ_{bi};

Type III: the elevation angle data set $S_E = \{\{E_i(t), t \in [t_i^c, T_i^c]\}, i = 1,2,\cdots,s_3\}$, which means the elevation angle of the spacecraft relative to the station with sampling interval h s in s_3 time periods, the i th tracking interval is $[t_i^c, T_i^c]$, the station coordinate is $X_{ci} = (x_{ci}, y_{ci}, z_{ci})$ and the transformation matrix is Φ_{ci};

Type IV: the range difference data set $S_P = \{\{P_i(t), t \in [t_i^d, T_i^d]\}, i = 1,2,\cdots,s_4\}$, which means the distance difference data of main station-spacecraft-assistant station with sampling interval h s in S_4 time periods, the i th tracking time is $[t_i^d, T_i^d]$, the corresponding main station coordinate is $X_{d0} = (x_{d0}, y_{d0}, z_{d0})$ and the assistant station coordinate is $X_{di} = (x_{di}, y_{di}, z_{di})$.

Based on the above classification, a new multi-source data fusion approach is applied to establish the trajectory coordinate fusion calculation method for the spacecraft during long-term flight with multi-station's link tracking. A vector function is established as

$$\begin{pmatrix} x_\kappa(X(t)) \\ y_\kappa(X(t)) \\ z_\kappa(X(t)) \end{pmatrix} = \Phi_\kappa^\tau \begin{pmatrix} x(t) - x_\kappa \\ y(t) - y_\kappa \\ z(t) - z_\kappa \end{pmatrix}, \quad \kappa \in \{b_1, \cdots, b_{s2}, c_1, \cdots, c_{s3}\} \tag{1}$$

where, Φ_κ^τ is the transformation matrix of b or c (azimuth angle or elevation angle) data source station $\tau \in \{1,2,\cdots,b_1 + \cdots + b_{s2} + c_1 + \cdots + c_{s3}\}$.

Denoting $D_\kappa(X) = \sqrt{(x - x_\kappa)^2 + (y - y_\kappa)^2 + (z - z_\kappa)^2}$ where $\kappa \in \{a_1, \cdots, b_{s1}, \cdots, d_1, \cdots, d_{s4}\}$ and to establish an objective function as following

$$\begin{aligned} F(X(t)) = & \sum_{i=1}^{s_1} \{R_i(t) - D_{ai}(X(t))\}^2 \phi(t, [t_i^a, T_i^a]) \\ & + \sum_{i=1}^{s_2} \left\{ \sqrt{x_{bi}(X(t)^2 + z_{bi}(X(t))^2} \sin A_i(t) - z_{bi}(X(t)) \right\}^2 \phi(t, [t_i^b, T_i^b]) \\ & + \sum_{i=1}^{s_3} \{D_{ci}(X(t)) \sin E_i(t) - y_{ci}(X(t))\}^2 \phi(t, [t_i^c, T_i^c]) \\ & + \sum_{i=1}^{s_4} \{P(t)_i - (D_{d0}(X(t) - D_{di}(X(t)))\}^2 \phi(t, [t_i^d, T_i^d]) \end{aligned} \tag{2}$$

where the switch function is defined as

$$\phi(s, [t, T]) = \begin{cases} 1, & s \in [t, T] \\ 0, & s \notin [t, T] \end{cases} \tag{3}$$

Sequence the tracking time period (from the beginning time to the ending time) in an order from short to long, and consider the constraint that calculable information shall be not less than 3 measurement units. Let's arrange the beginning time and ending time for all of the tracking equipment so as to find the least and the largest time point respectively

$$\{t_1^a, \cdots, t_{s1}^a; t_1^b, \cdots, t_{s2}^b; t_1^c, \cdots, t_{s3}^c; t_1^d, \cdots, t_{s4}^d\} \Rightarrow \{t_{(1)}^0, \cdots, t_{(s1+s2+s3+s4)}^0\} \tag{4}$$

$$\{T_1^a, \cdots, T_{s1}^a; T_1^b, \cdots, T_{s2}^b; T_1^c, \cdots, T_{s3}^c; T_1^d, \cdots, T_{s4}^d\} \Rightarrow \{T_{(1)}^0, \cdots, T_{(s1+s2+s3+s4)}^0\} \tag{5}$$

then determine the beginning calculation time point and the ending calculation time point

$$\begin{cases} t^0 = t_{(3)}^0 \\ t^e = T_{(s1+s2+s3+s4-3)}^0 \end{cases} \tag{6}$$

where $t_{(3)}^0$ is the 3rd time value after sequencing the beginning time and the ending time of each tracking equipment; $T_{(s1+s2+s3+s4-3)}^0$ is the 4th time value to last.

By using the multivariable non-linear function in formula (2), the minimum point can be get and the point-by-point calculation of the trajectory coordinate can be realized at each sampling moment within $[t^0, t^e]$ in the whole flight process.

$$\hat{X}(t) = \arg\min\{F(X)\} \tag{7}$$

In order to solve the multivariable non-linear function extreme value problem (7), the steepest descent method[10-11] is used in 9 steps as follows:

Step 1: properly select the initial value $X_0 = (x_\Theta(t), y_\Theta(t), z_\Theta(t))$, $k = 0$ and the threshold.

Step 2: calculate negative gradient $S_k = -\nabla F(X_k)$ and its unit vector $\hat{S}_k = -\nabla F(X_k) / \|\nabla F(X_k)\|$;

Step 3: if $\|S^k\| \le \alpha$ or $k \ge 1000$ (Set the maximum threshold of the iteration in order to prevent program's entering the infinite loop, set according to the calculation time and the calculation velocity), execute the 8th step; otherwise execute the 4th step;

Step 4: calculate the length $\rho_k = \|F(X_k) - F(X_{k-1})\|$;

Step 5: calculate $X_{k+1} = X_k + \rho_k \hat{S}_k$ and $F(X_{k+1})$;

Step 6: if $F(X_{k+1}) - F(X_k) \le \beta$, then execute the step 8; otherwise, execute the step 7;

Step 7: given $k+1 \Rightarrow k$, then execute the step 2;

Step 8: circulate the above process, complete the spacecraft coordinate calculation of each time point from t^0 to t^e and output location coordinate $X(t_i)$ of the spacecraft in launch coordinate system at each time point $t_i \in \{t^0, t^0 + h, \cdots, t^e\}$.

Step 9: output the location coordinate $X(t) = X_{k+1}$.

III. Fault-Tolerant Smoothing of the Spacecraft Trajectory

The trajectory algorithm described above can realize different type of data fusion during different time periods. However, since this algorithm is based on the least square (short as LS) theory, the calculation result will result partially abnormity or distortion from outliers which exists in the tracking data. In order words, the point-by-point LS calculation algorithm can not eliminate the fluctuation caused by measurement outliers, which make the calculation trajectory badly match with the practical one. Thus, this section will complete fault-tolerance improvement on formula (2). The squared loss function used in the least square algorithm are replaced with the attenuation function ρ described in Fig. 1, which is composed of the even function $\rho(x)$.

$$\begin{cases} \rho(x) = \begin{cases} \dfrac{c_1 c_2}{2}, & x > c_2 \\ g(x), & c_1 < x \le c_2 \\ \dfrac{x^2}{2}, & 0 \le x \le c_1 \end{cases} \\ g(x) = \dfrac{c_1(-x^2 + 2c_2 x - c_1 c_2)}{2(c_2 - c_1)} \end{cases} \tag{8}$$

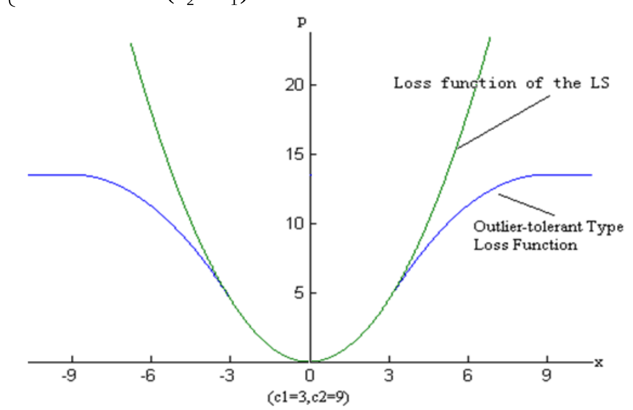


Fig. 1. Comparing between Least Square Loss Function and Fault-Tolerance Loss Function

Setting the window width $\Delta = kh$ ($k \in \{1, 2, \cdots, n\}$), the sliding fault-tolerance smooth is executed on the trajectory data. The spacecraft trajectory result disturbance caused by random error is weakened and the calculation result distortion caused by outliers of measurement data is eliminated. To apply the loss function in formula (8), the fault-tolerance estimation method based on the coefficient of cubic fitting curve $\theta_\omega = (\alpha_\omega, \beta_\omega, \gamma_\omega, \tau_\omega)$ are established as follows

$$\begin{pmatrix} \hat{\alpha}_\omega \\ \hat{\beta}_\omega \\ \hat{\gamma}_\omega \\ \hat{\tau}_\omega \end{pmatrix} = \arg \underset{\alpha,\beta,\gamma,\tau}{\min izing} \left\{ \sum_{t=t_i-\Delta}^{t_i+\Delta} \rho \left(\frac{\omega(t) - (\alpha + \beta t + \gamma t^2 + \tau t^3)}{\sqrt{\frac{1}{2k+1} \sum_{t=t_i-\Delta}^{t_i+\Delta} \left(\omega(t) - (\alpha + \beta t + \gamma t^2 + \tau t^3) \right)^2}} \right) \right\} \tag{9}$$

where $\omega \in \{x, y, z\}$, $\alpha_\omega, \beta_\omega, \gamma_\omega, \tau_\omega$ are respectively the minimums of $\alpha, \beta, \gamma, \tau$ .From formula (9), fault-tolerant estimation in the sliding window can be get.

$$\begin{pmatrix} \hat{x}(t) \\ \hat{y}(t) \\ \hat{z}(t) \end{pmatrix} = \begin{pmatrix} \hat{\alpha}_x + \hat{\beta}_x t + \hat{\gamma}_x t^2 + \hat{\tau}_x t^3 \\ \hat{\alpha}_y + \hat{\beta}_y t + \hat{\gamma}_y t^2 + \hat{\tau}_y t^3 \\ \hat{\alpha}_z + \hat{\beta}_z t + \hat{\gamma}_z t^2 + \hat{\tau}_z t^3 \end{pmatrix} \tag{10}$$

It is easy to validate, due to the stability and continuity of time, the trajectory coordinate in formula (10) only relates to time, and will be hardly influenced by data outliers or random error disturbance, which reflects the operation state of the spacecraft more precisely.

IV. Fault-Tolerance Calculation of Spacecraft Attitude Parameter

The common parameters to describe the space attitude of plane or other spacecraft are pitch angle, yaw angle and rolling angle. Assuming that the direction of longitudinal axis of the spacecraft is identical with the tangent line of trajectories, the calculation result can be applied in formula (10) of trajectory algorithm and a simplification algorithm of space craft attitude parameters can be established through numerical differentiation.

The 1st step is to execute sliding polynomial fault-tolerance differential flatness on the spacecraft trajectory coordinate $\{(x(t_i), y(t_i), z(t_i)) \mid t_i = t^0, t^0 + h, \cdots, t^c\}$ .So, the spacecraft's flight velocity is determined in the measurement range of TT&C network at any time.

According to $(\hat{\alpha}_\omega, \hat{\beta}_\omega, \hat{\gamma}_\omega)$ obtained during the calculation of coordinate components $w \in \{x, y, z\}$, the fault-tolerance estimation is performed to calculate velocity $(\dot{x}(t), \dot{y}(t), \dot{z}(t))$ in the measurement range of TT&C network at any $t \in [t^0 + 5h, t^e - 4h]$ as

$$\begin{pmatrix} \hat{\dot{x}}(t) \\ \hat{\dot{y}}(t) \\ \hat{\dot{z}}(t) \end{pmatrix} = \begin{pmatrix} \hat{\alpha}_x & \hat{\beta}_x & \hat{\gamma}_x & \hat{\tau}_x \\ \hat{\alpha}_y & \hat{\beta}_y & \hat{\gamma}_y & \hat{\tau}_y \\ \hat{\alpha}_z & \hat{\beta}_z & \hat{\gamma}_z & \hat{\tau}_z \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 2t \\ 3t^2 \end{pmatrix} \tag{11}$$

When $t_i \le t^0 + 5h$, $\theta_\omega = (\alpha_\omega, \beta_\omega, \gamma_\omega, \tau_\omega)^\tau$ can be replaced in formula (11) with the calculation result of formula (12) (θ_ω and w can be referenced in formula (9) and (10)).

$$\begin{pmatrix} a_0^w \\ \vdots \\ a_3^w \end{pmatrix} = \left\{ \begin{pmatrix} (t^0)^0 & \cdots & t_{i+5}^0 \\ \vdots & & \vdots \\ (t^0)^3 & \cdots & t_{i+5}^3 \end{pmatrix} \begin{pmatrix} (t^0)^0 & \cdots & (t^0)^3 \\ \vdots & & \vdots \\ t_{i+5}^0 & \cdots & t_{i+5}^3 \end{pmatrix} \right\}^{-1} \begin{pmatrix} (t^0)^0 & \cdots & t_{i+5}^0 \\ \vdots & & \vdots \\ (t^0)^3 & \cdots & t_{i+5}^3 \end{pmatrix} \begin{pmatrix} w(t_0) \\ \vdots \\ w(t_{i+5}) \end{pmatrix} \tag{12}$$

When $t_i \ge t^e - 4h$, $\theta_\omega = (\alpha_\omega, \beta_\omega, \gamma_\omega, \tau_\omega)^\tau$ can be replaced in formula (11) with the result form formula (13).

$$\begin{pmatrix} a_0^w \\ \vdots \\ a_3^w \end{pmatrix} = \left\{ \begin{pmatrix} t_{i-5}^0 & \cdots & (t^e)^0 \\ \vdots & & \vdots \\ t_{i-5}^3 & \cdots & (t^e)^3 \end{pmatrix} \begin{pmatrix} t_{i-5}^0 & \cdots & t_{i-5}^3 \\ \vdots & & \vdots \\ (t^e)^0 & \cdots & (t^e)^3 \end{pmatrix} \right\}^{-1} \begin{pmatrix} t_{i-5}^0 & \cdots & (t^e)^0 \\ \vdots & & \vdots \\ t_{i-5}^3 & \cdots & (t^e)^3 \end{pmatrix} \begin{pmatrix} w(t_{i-5}) \\ \vdots \\ w(t^e) \end{pmatrix} \tag{13}$$

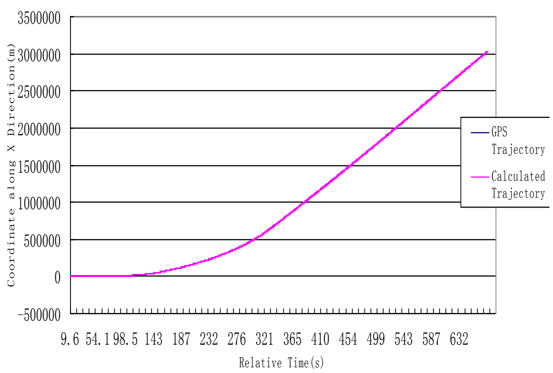
Circularly calculate the velocity parameters at each time point $t_i \in \{t^0, \cdots, t^e\}$

The 2nd step, according to formula (9)-(13), if the direction of longitudinal axis of the spacecraft is identical with the tangent line of trajectory, calculate the pitch angle and yaw angle:

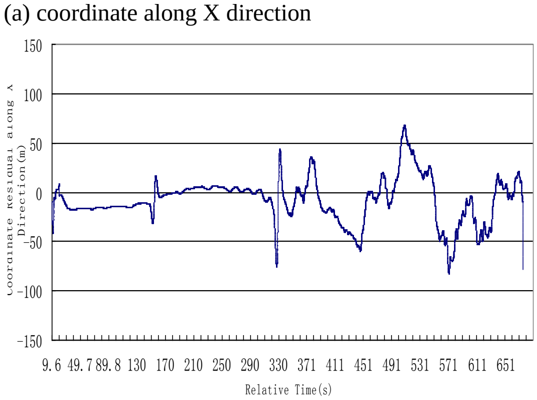
$$\theta = \arcsin\left(\frac{\hat{\dot{y}}}{\sqrt{\hat{\dot{x}}^2 + \hat{\dot{y}}^2}}\right), \ \sigma = \arcsin\left(\frac{-\hat{\dot{z}}}{\sqrt{\hat{\dot{x}}^2 + \hat{\dot{y}}^2 + \hat{\dot{z}}^2}}\right) \tag{14}$$

V. Practical Applications

This paper applies measurement data of some spacecrafts from some equipments (3 single pulse radars, 1 USB device and 4 multiple velocity measurement system) to validate and the results are shown in following figures. The blue line is theoretical trajectory, and the pink line is the calculated trajectory.

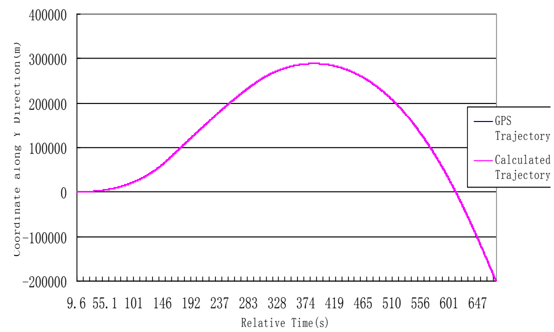


(a) coordinate along X direction

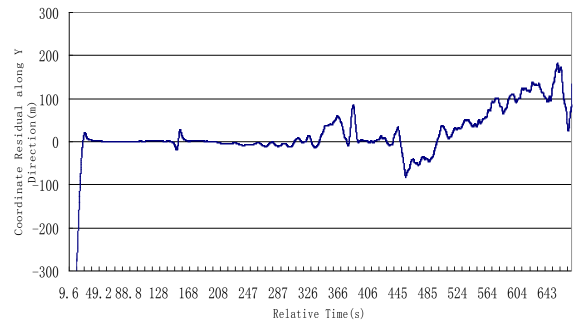


(b) coordinate residual along X direction

Fig. 2. coordinate contrastive diagram along X direction

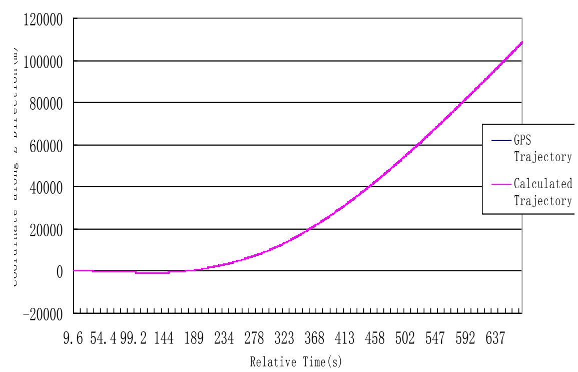


(a) coordinate along Y direction

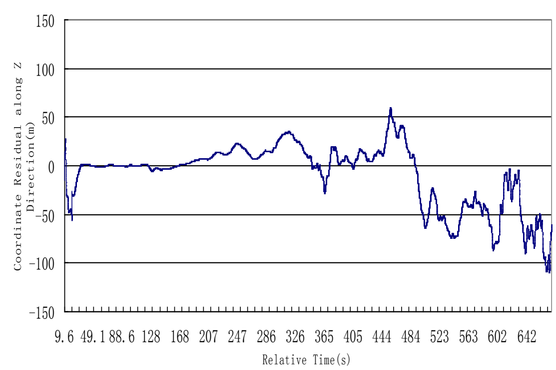


(b) coordinate residual along Y direction

Fig. 3. coordinate contrastive diagram along Y direction

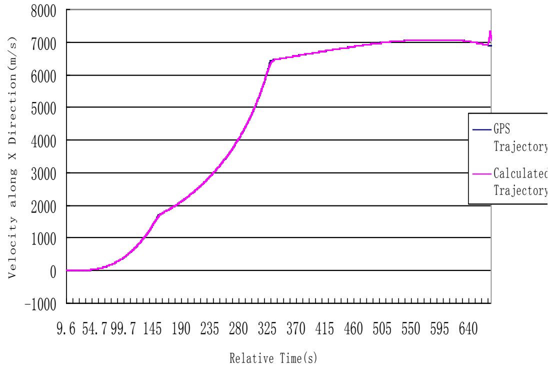


(a) coordinate along Z direction

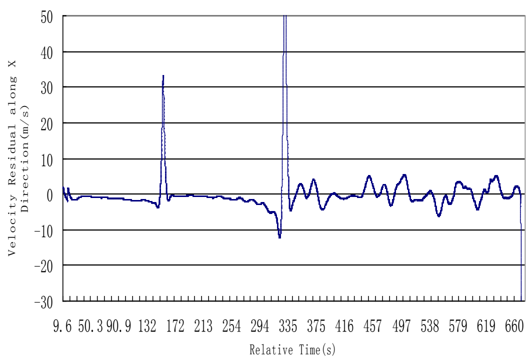


(b) coordinate residual along Z direction

Fig. 4. coordinate contrastive diagram along Z direction

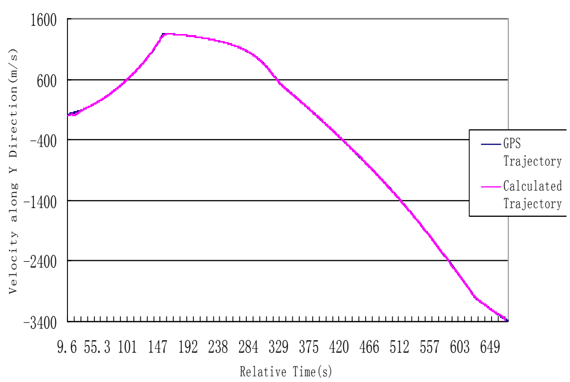


(a) velocity along X direction

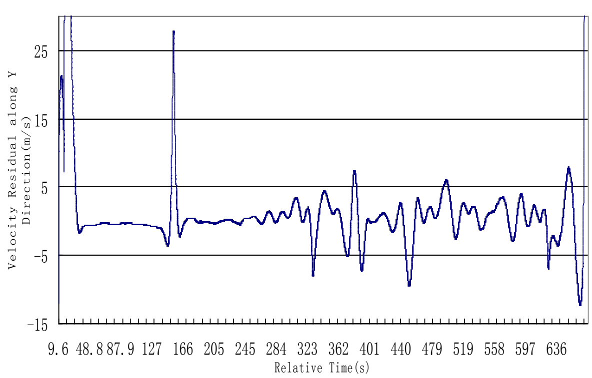


(b) velocity residual along X direction

Fig. 5. velocity contrastive diagram along Z direction

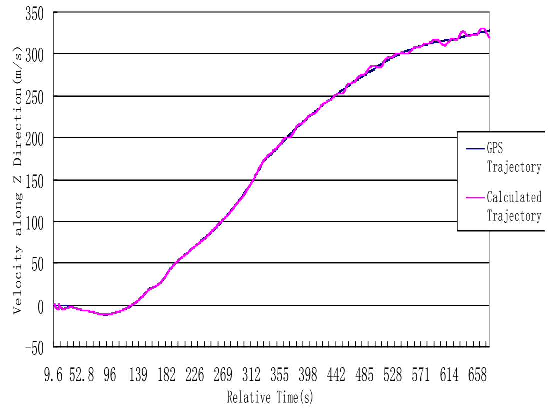


(a) velocity along Y direction

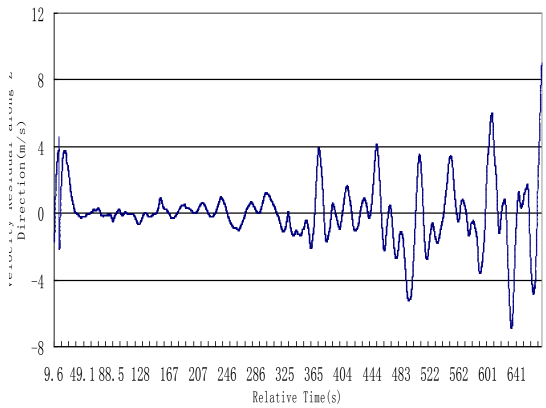


(b) velocity residual along Y direction

Fig. 6. velocity contrastive diagram along Y direction

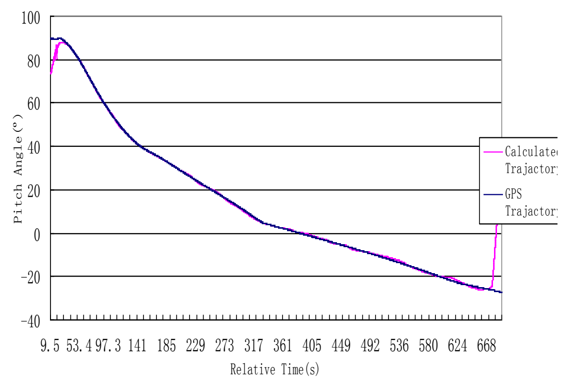


(a) velocity along Z direction

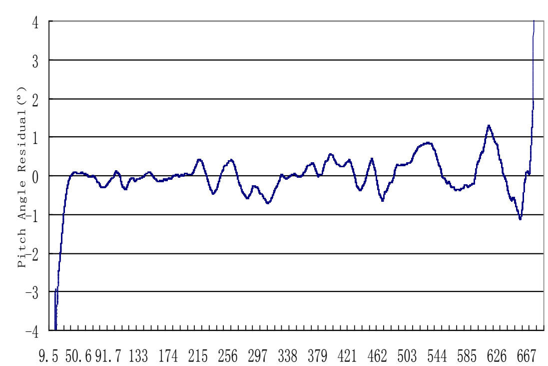


(b) velocity residual along Z direction

Fig. 7. velocity contrastive diagram along Z direction

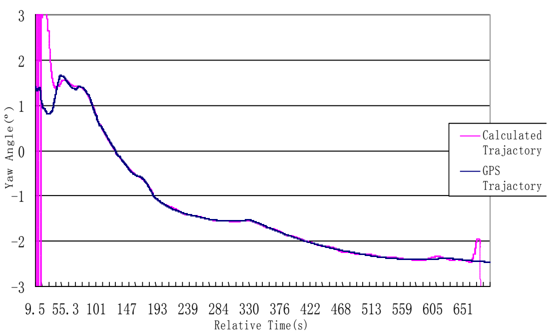


(a) pitch angle

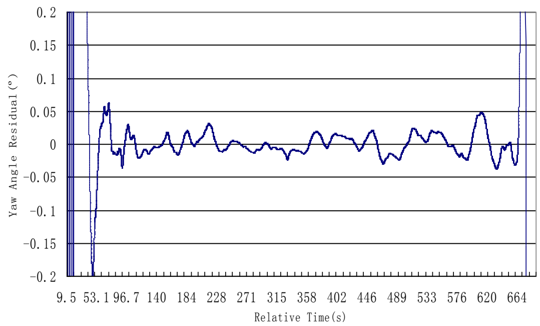


(b) pitch angle residual

Fig. 8. pitch angle contrastive diagram of spacecraft



(a) yaw angle



(b) yaw angle residual

Fig. 9. yaw angle contrast diagram of spacecraft

Fig. 2 ~9 show out the effects which the trajectory computed by this paper's method contrasts with the theoretical trajectory. It can be perceived that this paper's method can compute a whole and continuous trajectory, and this trajectory without distortion and deformation demonstrates the reliability of this paper's method.

VI. Conclusion

In both aeronautic and astronautic field, the monitoring and navigation of the spacecraft is realized by using multiple tracking measurement equipments through partial overlapped relay tracking mode. How to take full advantage of measurement data from multiple TT&C equipments to realize the accurate calculation in tracking trajectory and attitude, which is a technological subject mainly concerned in spacecraft navigation and flight performance analysis field.

This paper establishes a flight trajectory and flight attitude parameter multi-source data fault-tolerance fusion algorithm based on multi-variable non-linear function of the extreme value steepest descent method. This algorithm can rapidly and reliably calculate the trajectory and attitude of a spacecraft in multi- equipment overlapped tracking mode. It can take full advantage of effective data from different equipments, effectively avoid the bad influence of data outliers without data outlier detecting and repair, and remarkably improve the consistency and reliability of spacecraft's trajectory and attitude calculation result without piecewise calculation according to equipments. According to Fig. 1~8, the trajectory obtained by using the algorithm proposed by this paper is complete and continuous without distortion, which proves the reliability of this algorithm.

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References

[1] LU Li-sheng, ZHANG Yu-xiang, LI Jie. External Trajectory Measurement Data Processing. Beijing: National Defence Industry Press, 2002,pp:270-366

[2] Hu Shao-lin, XU Ai-hua, GUO Xiao-hong. Technique of Processing Pulse Radar Measurement Data. Beijing: National Defence Industry Press, 2007, pp:120-139

[3] CUI Shu-hua, HU Shao-lin. Technique of Optical Tracking Measurement Data Processing. Beijing: National Defence Industry Press, 2014,pp:26-107

[4] H Durrant-Whyte, T C Henderson. Multisensor Data Fusion. In Springer Handbook of Robotics,Springer Press,2008,585-610

[5] M E Liggins,D L Hall,J Llinas.Handbook of Multisensor Data Fusion:Theory and Practice. Taylor & Francis Group,CRC Press, 2009,1-44

[6] J R Raol.Multi-Sensor Data Fusion with Mathlab.Taylor & Francis Group,CRC Press, 2009,1-568.

[7] P Kmiotek. Multi-sensor Data Fusion for Representing and Tracking Dynamics Objects. Disseration of PhD,Universite de Technologe de Delfort-Montbeliard, Karkow,2009,pp:5-41 & 125-156.

[8] Shaolin Hu,Xiaofeng Wang,Karl Meinke,etal.Outlier-tolerant Fitting and Online Diagnosis of Outliers in Dynamic Process Sampling Data Series. Proc of 3rd International Conference of Artificial Intelligence and Computational Intelligence, Lecture Notes in Computer Science, vol7004,2011,pp:195-203

[9] LI Hong-yi. Idealized Steepest Descent Method and Its Approximate Example. [J] China: Journal of Shanghai Second Polytechnic University, 2011,28(1):8-13

[10] LIU Ang-ran. Iterative Solution of Linear Equation and Steepest Descent Method [J] China: Journal of Chifeng University(Natural Science Edition), 2014,30(2):10-13.