

# Peak-to-Average Power Ratio Reduction based Varied Phase for MIMO-OFDM Systems

Lahcen Amhaimar<sup>1</sup>, Saida Ahyoud<sup>2</sup>  
and Adel Asselman<sup>1</sup>

<sup>1</sup>Optics and Photonics Team

<sup>2</sup>Information Technology and Systems Modeling Team

Abdelmalek Essaadi University,

Faculty of Science

Tetuan, Morocco

Elkhaldi Said

Electronic and Instrumentation Team,

Abdelmalek Essaadi University,

Faculty of Science

Tetuan, Morocco

**Abstract**—One of the severe drawbacks of orthogonal frequency division multiplexing (OFDM) is high Peak-to-Average Power Ratio (PAPR) of transmitted OFDM signals. During modulation the sub-carriers are added together with same phase which increases the value of PAPR, leading to more interference and limits power efficiency of High Power Amplifier (HPA), it's requires power amplifier's (PAs) with large linear operating ranges but such PAs are difficult to design and costly to manufacture. Therefore, to reduce PAPR various methods have been proposed. As a promising scheme, partial transmit sequences (PTS) provides an effective solution for PAPR reduction of OFDM signals. In this paper, we propose a PAPR reduction method for an OFDM system with variation of phases based on PTS schemes and Solid State Power Amplifiers (SSPA) of Saleh model in conjunction with digital predistortion (DPD), in order to improve the performance in terms of PAPR, the HPA linearity and for the sake of mitigating the in-band distortion and the spectrum regrowth. The simulation results show that the proposed algorithm can not only reduces the PAPR significantly, but also improves the out-of-band radiation and decreases the computational complexity.

**Keywords**—OFDM; MIMO; PAPR; PTS; HPA; GA

## I. INTRODUCTION

A combination of multiple-input multiple-output (MIMO) with orthogonal frequency division multiplexing (OFDM) has lead to significant advancement in wireless communication systems. It has been receiving a great deal of attention as a solution of high-quality service for next generation. OFDM systems has been adopted by several emerging wireless applications, such as WiMAX and digital video broadcasting/digital audio broadcasting (DVB/DAB) [1], thanks to its robustness over frequency selective fading channels and high bandwidth efficiency [2]. Still, some challenging issues remain unresolved in the design of the OFDM systems such as Peak to Average Power Ratio (PAPR) in the transmission system. The high PAPR reduces the efficiency of OFDM systems by introducing the intermodulation distortion and undesired out-of-band radiation, due to, the nonlinearity of the high power amplifier (HPA). Thus, it is highly desirable to improve PAPR performance of the signal. In the literature, many PAPR reduction techniques have been proposed and each of them has their own advantages and disadvantages [3], such as interleaving [4], clipping, companding [5], selective mapping

(SLM) [6], [7], partial transmit sequence (PTS) [8]–[10], tone reservation [11], coding technique [12], adaptive mode with low complexity [13], signal set expansion [14], and active constellation expansion [15].

This paper investigates the performance of PAPR reduction in OFDM and MIMO-OFDM system. An optimization with the same phase weighting will be proposed, and throughput results are presented for IEEE 802.11a and IEEE 802.16 standard.

The remainder of this paper is organized as follows. Section 2, present the basic concept of OFDM system, such as OFDM signals, definition and measurement of PAPR of OFDM signal, and PTS algorithm is going to be showed in this study. Section 3 introduces the principles of the proposed system. In Section 4, the performances of proposed method are discussed, and the simulated results are going to be stated. Finally, in section 5, a conclusion is drawn.

## II. CHARACTERISTICS OF OFDM SIGNALS

### A. PAPR in OFDM Signals

In OFDM system, the baseband operations at the transmitter include mapping the information data bit stream to symbols according to a certain modulation scheme, such as M-PSK or M-QAM, to create a vector of complex-valued symbols,  $X = [X_0, X_1, \dots, X_{N-1}]^T$ . The data streams transmitted simultaneously by subcarriers. Each of the sub-carriers is independently modulated and multiplexed. Then, an OFDM signal is formed by summing all the N modulated independent subcarriers which are of equal bandwidth. The IFFT generates the ready-to-transmit OFDM signal. The sub-carriers are chosen to be orthogonal so that the adjacent sub-carriers can be separated. In the discrete time domain, the mathematical representation of baseband OFDM signal  $x[n]$  can be expressed as

$$x[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j\frac{2\pi nk}{LN}}, \quad 0 \leq n \leq LN - 1 \quad (1)$$

where  $X_k$  is the complex symbol transmitted on the kth subcarrier, N is the size of the IFFT and L is the oversampling factor.

The PAPR of an OFDM signal is defined as the ratio of the maximum to the average power of the signal, as follows

$$PAPR\{x[n]\} = \frac{\max\{|x[n]|^2\}}{E\{|x[n]|^2\}}, 0 \leq n \leq LN - 1 \quad (2)$$

where  $E\{\cdot\}$  denotes the expected value operation.

The PAPR for the discrete-time baseband signal  $x[n]$  may not be the same as that for the continuous time baseband signal  $x(t)$ . In fact, the PAPR for  $x[n]$  is lower than that for  $x(t)$ , simply because  $x[n]$  may not have all the peaks of  $x(t)$ . Fortunately, to avoid this problem, the oversampling is usually employed. It is shown in [3], [16], that an oversampling factor  $L=4$  is sufficient to approximate the real PAPR results.

The complementary cumulative distribution function (CCDF) of PAPR provides information about the percentage of OFDM signals that have PAPR above a particular level. It denotes the probability that the PAPR of an OFDM symbol exceeds the given threshold  $PAPR_0$ , which can be expressed as

$$CCDF(N, PAPR_0) = Pr\{PAPR > PAPR_0\} \quad (3)$$

### B. PAPR in MIMO-OFDM

We define the PAPR of a MIMO-OFDM signal as the maximum of the PAPRs among all the parallel transmit antenna branches. PAPR at the one transmit antenna is defined as the ratio of the peak power to the average power of an OFDM signal in that branch. The MIMO-OFDM PAPR system can be expressed as [17]

$$PAPR_{MIMO} = \max PAPR_i, i = 1, \dots, N_T \quad (4)$$

where  $N_T$  is the number of transmission antennas.

### C. PTS for PAPR Reduction

The block diagram of OFDM transmitter with partial transmit sequence (PTS) technique is shown in Fig.1 [8]. The PTS approach partitions an input data block of  $N$  complex symbols into  $V$  disjoint sub-blocks as follows

$$X = [X^0, X^1, X^2, \dots, X^{V-1}]^T \quad (5)$$

where the sub-carriers in each sub-block are weighted by phase rotations, and all the subcarriers which are occupied by the other sub-blocks are set to zero. The various partitioning methods to divide complex symbols into disjoint sub-blocks are proposed in [18], these include pseudorandom, adjacent and interleaved partitioning schemes. In PTS, shown in Fig. 1 each partitioned sub-block is multiplied by a corresponding complex phase factor to produce the sequences

$$X = \sum_{v=1}^V X^v b^v, b^v = e^{j\varphi^v}, v = 1, 2, \dots, V \quad (6)$$

Afterward taking its IFFT to yield [8], [10]

$$x = IFFT \left\{ \sum_{v=1}^V b^v X^v \right\} = \sum_{v=1}^V b^v \cdot IFFT\{X^v\} = \sum_{v=1}^V b^v x^v \quad (7)$$

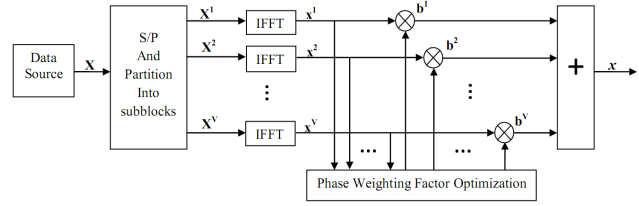


Fig. 1: Block diagram of PTS technique

where  $x$  denotes the candidate sequence.

The phase vector is chosen so that the PAPR can be minimized [8], which is shown as

$$[b^1, \dots, b^V] = \arg \min_{[b^1, \dots, b^V]} \left( \max_{n=0,1,\dots,N-1} \left| \sum_{v=1}^V b^v x^v[n] \right| \right) \quad (8)$$

In the practical application of PTS, a set of phase weighting factors is usually selected for generating phase weighting sequences. To assume that there are  $W$  allowed phases weighting factors in this set. Without any loss of performance, we can set phase weighting factor for the first sub-block to one and observe that there are  $(V-1)$  sub-blocks to be optimized. To match the optimal phase weighting sequence for each input data sequence,  $W^{V-1}$  possible combinations should be checked, and then the candidate sequence with the minimum PAPR is selected for transmitting [8], [19].

### III. PROPOSED APPROACH FOR PAPR REDUCTION

In this Section we are going to describe the proposed method, it is based on combining signal sub-blocks which are phased shifted by the same phase factors to generate multiple candidate signals, so as to select the ideal PAPR signal. The steps involved in the generation and process of phase weighting factors are summarized as follows:

- As a first step, we find the PAPR of original signal and set it as  $PAPR_{min}$
- Then, the input data sequence is partitioned into  $V$  sub-blocks  $X_{(v)}$  as in equation (5).
- Next, Then, we set the search space of phase weight factor ( $angle[0, 2\pi]$ ) as  $(0^\circ, 10^\circ, 20^\circ, \dots, 360^\circ)$  with an increment of  $10^\circ$ , and start the optimization of each sub-block with the same phase factor  $\varphi$ .

$$X_{(v)} \cdot \varphi, \text{ and } \varphi = e^{j\theta} \quad (9)$$

where  $v = 1, \dots, V$ . and  $\theta = 0^\circ, 10^\circ, 20^\circ, \dots, 360^\circ$

- After that, compute the PAPR of the combined signal  $x$  (Eq. 10). If  $PAPR > PAPR_{min}$ , switch  $\varphi$  to the next phase. Otherwise, update  $PAPR_{min} = PAPR$ .

$$x = IFFT \left\{ \sum_{v=1}^V X_{(v)} \cdot \varphi \right\} \quad (10)$$

- The algorithm continues in this fashion until all the 36 phase factor are searched. Then, we retain the signal

with the PAPR minimum and the set of optimal phase factors.

The proposed approach is used to find a suitable phase factor set that minimizes the PAPR in a transmitted signal without exhaustive search. It decreases the computational load of the PTS technique by searching a small piece of a set of possibilities instead of the whole set as in the classical technique. Finally, The transmitted signal  $x(t)$  can be linearly amplified by virtue of the predistorter, a technique that corrects the nonlinearity by compensate nonlinear distortion of the power amplifier.

#### IV. SIMULATION RESULTS

In this section, we numerically evaluate the performance of the PAPR reduction scheme by extensive simulations were performed using the CCDF. The systems is carried out based on IEEE 802.11a and IEEE 802.16 standard with  $N=64$  and  $N=256$  subcarriers successively.  $10^4$  and  $10^5$  OFDM symbols are randomly chosen for the simulation with QPSK modulation and set the oversampling factor  $L=4$  for selecting and estimating the signal with minimum PAPR. The simulation parameters are also documented in the Table 1

TABLE I: Simulation Parameters

Parameter	Standards	
	IEEE 802.11a	IEEE 802.16
FFT size	64	256
User carriers	52	200
Pilot carriers	4	8
Number of null/guard band subcarriers	12	56
Cyclic prefix or guard time	1/4, 1/8, 1/16, 1/32	
Modulation	QPSK, 3/4	
Oversampling Factor	L=4	
Number of subblocks	V = 2, 4, 6, 8	
Number of phase factors	W =4 {1, -1, j, -j}	
Channel bandwidth	3.5MHz	
Noise Channel	AWGN	
SNR	30dB	
Number of generations	G = 10	
Population	P=5	
Crossover rate	CR=1.0	
Mutation rate	MR=0.05	

##### A. PAPR Reduction

In Fig. 2, the CCDF of PAPR are obtained by simulation, in which  $10^5$  OFDM symbols are randomly generated, four sub-blocks ( $V=4$ ) are used and the sets of phase weighting factors are varied between  $[0, 2\pi]$ . The simulation results show that the proposed approach can reduces the PAPR significantly. It is shown in Fig. 2 that the PARR reduction achieved with the new algorithm was 6.6 dB compared with original OFDM (10.7 dB) when  $CCDF = 10^{-3}$ .

Figure 3 illustrate the effectiveness of the proposed scheme as the number of sub block varies. The results of the simulation are based on the transmission of randomly generated OFDM symbols. It is seen that the PAPR performance improves as the number of sub blocks increases with  $V= 2, 4, 6,$  and  $8$ . So, by increasing the number of sub-blocks, the proposed system requires low transmitted power.

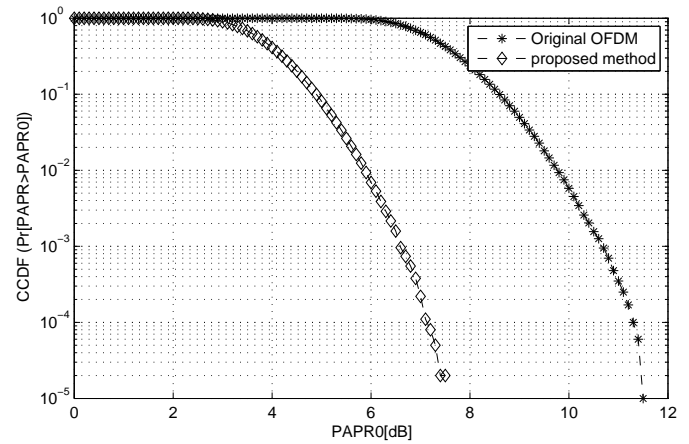


Fig. 2: PAPR reduction with proposed method

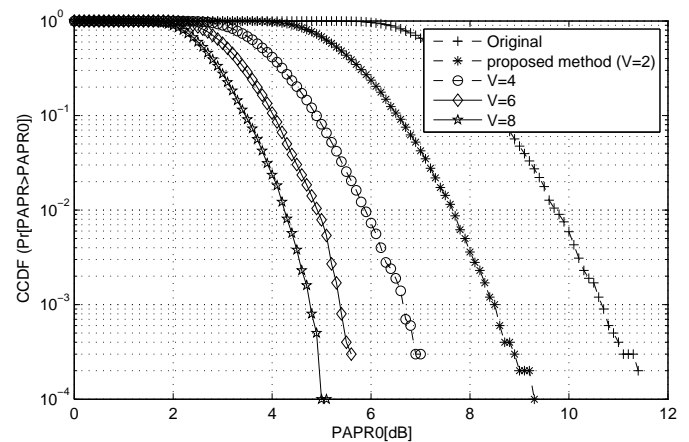


Fig. 3: PAPR performance with new technique when the number of sub-blocks varies

Next, figure 4 show the performance comparison between the proposed scheme and existing PTS and SLM in terms of CCDF. The curves of the simulations results are given for  $10^4$  OFDM symbols, in which four sub-blocks ( $V=4$ ) are used and the sets of phase weighting factors for PTS are  $\{1, -1, j, -j\}$  ( $W=4$ ).

From Fig.4, it is very clear that all schemes can reduce the PAPR largely in OFDM system. However, their performances of the PAPR reduction are different.

For example, when  $CCDF = 10^{-3}$ , the PAPRs are 5.23 dB, 6.6 dB, 7 dB and 10.7dB for the PTS, proposed scheme, SLM scheme and original OFDM signals, respectively. Although the PTS has better PAPR than the proposed technique, the computational load of the PTS is larger than our approach.

Finally, we consider PAPR reduction performance in MIMO-OFDM system. As shown in Fig. 5, the CCDF of the PAPR of original and optimized signals have been given for randomly generated QPSK symbols. MIMO-OFDM system uses two antennas based on Alamouti scheme with  $N=256$

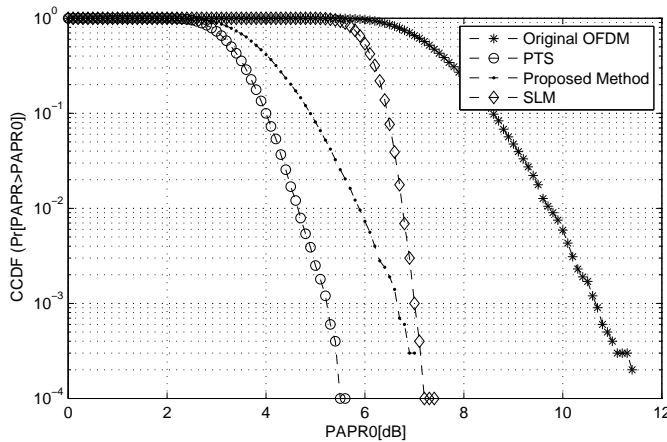


Fig. 4: Comparisons of CCDF based on different PAPR reductions schemes.

carriers per antenna, among those, 56 unused free carriers (as in IEEE802.16 WiMAX standard, Table 1). We set the oversampling factor  $L = 4$ . This approach achieves significant reduction in PAPR for MIMO OFDM systems. As we can see, the PAPR in transmitted signal is 7.5 dB for the proposed method compared with original MIMO-OFDM PAPR 11.25 dB at the  $10^{-3}$  probability level.

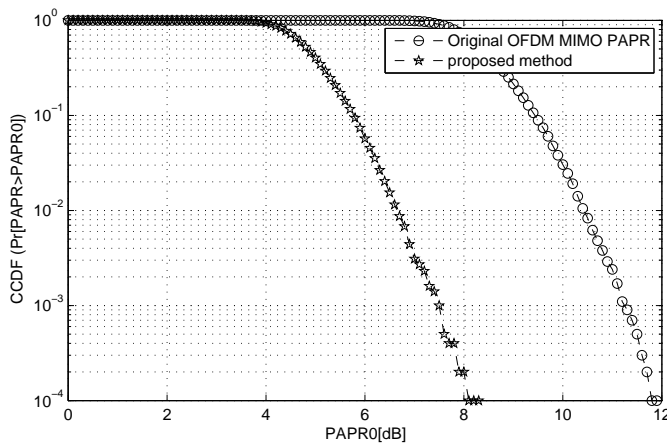


Fig. 5: The PAPR reduction performances in MIMO-OFDM systems

### B. Complexity Performances

Figure 6 shows the CCDF curves of OFDM system with different PAPR reduction techniques which are the proposed method, the original PTS technique, and the Genetic Algorithm with PTS technique (GA-PTS). The basic system parameters for the simulations are summarized in Table I. Although the PTS and the GA-PTS techniques have better PAPR than the proposed approach, the computational loads of these methods are larger than our method.

The GA is an optimization method serves as a solution to find a suitable phase factor set that minimizes the PAPR in

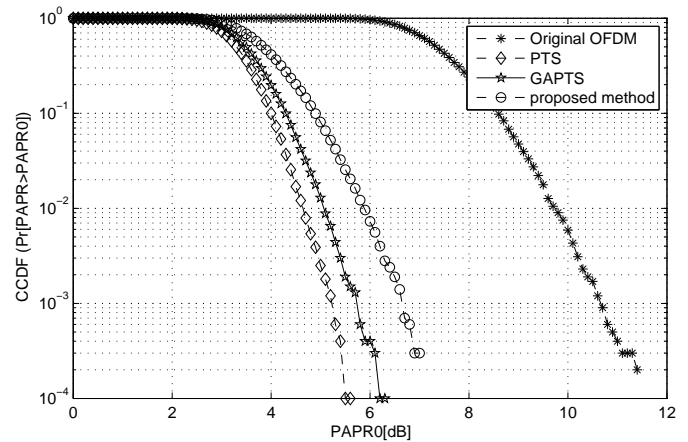


Fig. 6: Comparison of the  $PAPR_0$ (dB) versus CCDF in OFDM systems for proposed method, GA-PTS and original PTS

a transmitted signal [20], [21]. It decreases the computational load of the proposed technique by searching a small piece of a set of possibilities instead of the whole set as in the classical technique (PTS). The GA has a good convergence and high explorative ability. However that will result in higher complexity and slower convergence rate, usually we need a decent sized population and a lot of generations to guarantee the accuracy and explore the entire space, which takes more time for convergence.

Table II shows the number of search values for OFDM, using different schemes to find the phase factors. It is shown that when  $Pr(PAPR > PAPR_0) = 10^{-3}$ , the  $PAPR_0$  of the PTS is 5.23dB with exhaustive searching number  $W^V = 4^4 = 256$  (number of possible phase factor combinations). Values close to PTS scheme can be obtained by GA-PTS with  $P \times G = 5 \times 10 = 50$  searches, where  $P$  is the maximum size of the population and  $G$  is the generation. while proposed approach is 6.64dB with a small computational load, 36 iteration.

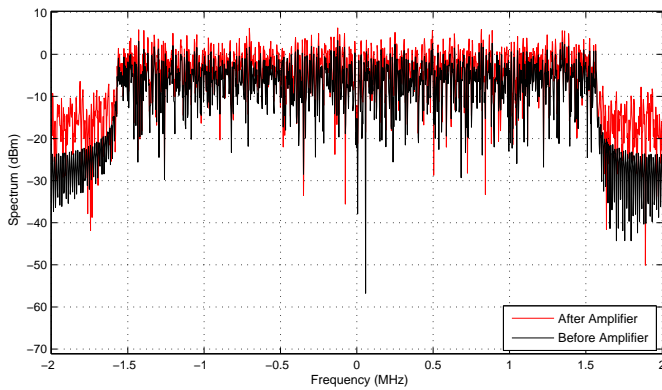
TABLE II: Computational Complexity of the Different Methods for  $CCDF = 10^{-3}$

Method ( $V=4, W=4$ )	Number of search	PAPR (dB)
Original OFDM	0	10.7
PTS	$W^V = 4^4 = 256$	5.23
GA-PTS	$P \times G = 50$	5.7
Proposed method	36	6.64

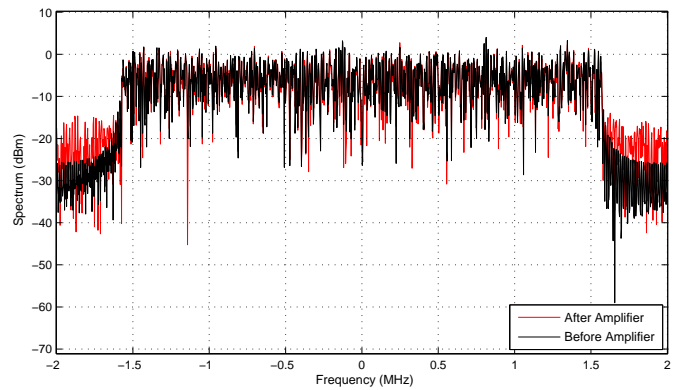
As a result, the proposed method with 36 searches was only 1.4 dB and 0.9dB higher than optimum PTS method and GA-PTS respectively, in OFDM systems. On the contrary, our approach has a low search complexity when compared with each method alone.

### C. Spectrum Performances

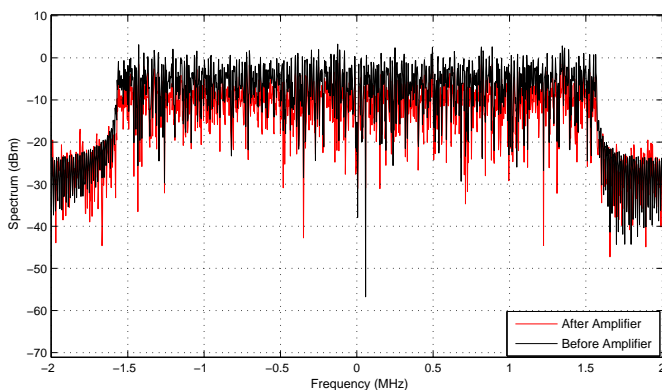
This analysis is carried out in the presence of two models of high power amplifier (HPA), are Solid State Power Amplifiers (SSPA) of Saleh model [22] and Rapp model [23] in conjunction with digital predistortion (DPD) as linearization technique.



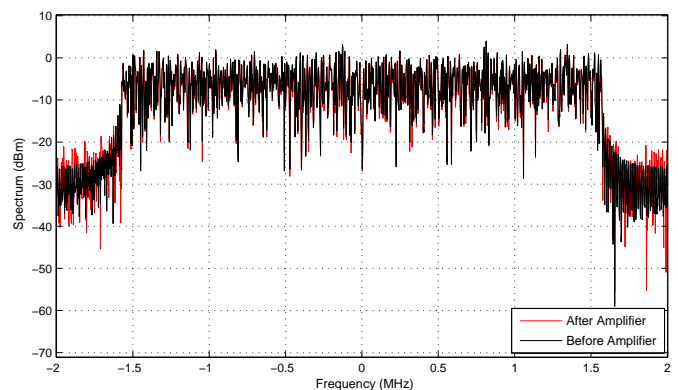
(a) Without PAPR reduction



(b) PAPR reduction with DPD



(c) Without PAPR reduction



(d) PAPR reduction with DPD

Fig. 7: Spectrum performance comparison of an OFDM signal with and without proposed method with severe nonlinearity. (a,b) saleh model and (c,d) rapp model.

Fig.7 show the signal of spectrum performance before and after passing through HPA model for conventional OFDM system and OFDM system with PAPR reduction method and DPD. The results are obtained for 1dB below the input power that causes amplifier saturation.

Figure 7.a and fig.7.b shows the frequency spectra when the HPA is working near the saturation region based saleh model. It is clearly seen that the proposed method with DPD has the ability to reduce the spectral regrowth around each of the two bands, the transmitted spectrum are kept below -15 dBm.

Next, fig.7.c and fig.7.d shows the spectrum with and without proposed technique with Rapps solid state power amplifier model, Comparing the two spectra, we can view a few change on spectrum due to characteristics of rapp model, it does not apply a phase change to the input signal. Therefore, the HPA power efficiency will be much higher in this case compared with saleh model.

## V. CONCLUSION

In this paper, a phase weighting method based PTS scheme with low computational complexity is proposed followed by a

predistortion technique in order to reduce the PAPR, improves the spectrum efficiency and the PA's linearity simultaneously. The sets of phase weighting factors are varied between  $[0, 2\pi]$  to search for a good set of phase factors, and the optimization was by one phase to obtain the desirable PAPR reduction. The simulation results show that the performances given by our approach, the PAPR reduction is better than original OFDM and maintained close to PAPR values of the PTS and G-PTS technique while providing a low computational load. We have also shown that our proposed method provides a good PAPR reduction in MIMO-OFDM at the transmitter. Moreover, the joint scheme using our approach with DPD decreased the transmitted spectrum regrowth at 1dB HPA backoff below amplifier saturation point. That's means when the HPA is working near or in the saturation region, the proposed PAPR reduction helps to improve the HPA efficiency and linearization.

## REFERENCES

- [1] Amhaimar, L., Ahyoud, S. and Asselman, A. "PAPR reduction performance for WiMAX OFDM systems;" In Proceeding IEEE of The Third International Workshop on RFID And Adaptive Wireless Sensor Networks, DOI: 10.1109/RAWSN.2015.7173274, 2015.

- [2] Wu, Y., and Zou, W. Y. "Orthogonal frequency division multiplexing: A multi-carrier modulation scheme," IEEE Transaction on Consumer Electronics, Vol. 41, 392-399,1995
- [3] Jiang, T. and Wu, Y. "An overview: Peak-to-average power ratio reduction techniques for OFDM signals," IEEE Transaction on Broadcasting, Vol. 54, 257-268, 2008.
- [4] Sakran, H., Shokair, M. and Elazm, A. A., "An efficient technique for reducing PAPR of OFDM system in the presence of nonlinear high power amplifier," Progress In Electromagnetics Research C, Vol. 2, 233-241, 2008.
- [5] Sakran, H., Shokair, M. and Elazm, A. A. "Combined interleaving and companding for PAPR reduction in OFDM systems," Progress In Electromagnetics Research C, Vol. 6, 67-78, 2009.
- [6] Bauml, R.W, Fischer, R.F.H, and Huber, J.B. "Reducing the peak-to-average power ratio of multicarrier modulation by selected mapping," Electron. Lett., Vol. 32, No. 22, 2056-2057, 1996
- [7] Ying, Liang, H. "Integrating CE and modified SLM to reduce the PAPR of OFDM systems," Wireless Personal Communication. doi:10.1007/s11277-014-2036-0, 2015
- [8] Muller, S.H. and Huber, J. B., "OFDM with reduced peak-to-average power ratio by optimum combination of partial transmit sequences," IEE Electronics Letters, Vol. 33, No. 5, 368-69, 1997.
- [9] Cimini, L.J., and Sollenberger, N. R., "Peak-to-Average Power Ratio Reduction of an OFDM Signal Using Partial Transmit Sequences," IEEE Commun. Lett, Vol. 4, No. 3, 86-88, 2000.
- [10] Varahram, P., Mohammady, S., and Ali, B. M. "A robust peak-to-average power ratio reduction scheme by inserting dummy signals with enhanced partial transmit sequence in OFDM systems," Wireless Personal Communications,72(2), 1125-1137, 2013.
- [11] Yang, M., and Shin, Y., "PAPR reduction using tone reservation for NC-OFDM transmissions," Ubiquitous and Future Network (ICUFN). In 4th International Conference, 454-455, 2012.
- [12] Jones, AE., Wilkinson, TA. and Barton, SK., "Block coding scheme for reduction of peak to mean envelope power ratio of multicarrier transmission schemes," Electron Lett, Vol. 30, No. 25, 2098-2099, 1994.
- [13] Rajbanshi, R., Wyglinski, A. M., & Minden, G. J. "Adaptive-mode peak-to-average power ratio reduction algorithm for OFDM-based cognitive radio." In Proceeding IEEE 64th Vehicle Technology Conference, Vol. 6, pp. 1350-1354, 2006.
- [14] Han, S. H. and Lee, J. H., "Peak-to-average power ratio reduction of an OFDM signal by signal set expansion," IEEE International Conference Communication, Vol. 2, 867-871, 2004.
- [15] Zhou, Y., and Jiang, T., "A novel clipping integrated into ACE for PAPR reduction in OFDM systems," In International Conference on Wireless Communications and Signal Processing, 1-4, 2009.
- [16] Tellado J, "Peak to average power reduction for multicarrier modulation," Ph.D. Thesis, Stanford University, Calif, USA, September 1999.
- [17] Fischer, R.F.H., Hoch, M., "Peak-to-average power ratio reduction in MIMO OFDM," IEEE International Conference on Communications (ICC 2007), Glasgow, United Kingdom, pp. 762-767, June 2007.
- [18] Kang, S.G., Kim, J.G. and Joo, E.K., "A novel sub-block partition scheme for partial transmits sequence OFDM," IEEE Trans. Broadcast. 45, pp. 333-338, 1999.
- [19] S.H. Han and J.H. Lee, "An overview of peak-to-average power ratio reduction techniques for multicarrier transmission, IEEE Wireless. Communications, vol. 12, no. 2, pp. 56-65, April 2005.
- [20] Lixia, M, Murrioni, M, Popescu, V, "PAPR reduction in multicarrier modulations using genetic algorithms," 12th International Conference on Optimization of Electrical and Electronic Equipment; Brasov, Romania. New York, NY, USA: IEEE. pp. 938-942, May 2010.
- [21] Liang, H, Chen, Y, Huang, Y, Cheng, C, "A modified genetic algorithm PTS technique for PAPR reduction in OFDM systems," In: 15th Asia-Pacific Conference on Communications; pp. 182-185, 10 October 2009, Shanghai, China. New York, NY, USA: IEEE.
- [22] ADEL, A, Saleh, M, "Frequency-independent and frequency-dependent non-linear models of TWT amplifiers," IEEE Trans. Commun., vol. 29, No. 11, 1715-1720,1981.
- [23] Rapp, C., "Effects of HPA-Nonlinearity on a 4-DPSK/OFDM-Signal for a Digital Sound Broadcasting System," in Proceedings of the Second European Conference on Satellite Communications, Liege, Belgium, pp. 179-184, Oct. 22-24 .1991