A Novel Algorithm to Improve Resolution for Very Few Samples

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Abstract—**This paper presents a new technic to improve resolution and direction of arrival (DOA) estimation of two closed source, in array processing, when only few samples of received signal are available. In these conditions, the detection of sources (targets) is more arduous, and even breaks down. To overcome these problems, a new algorithm is proposed. It combines spatial smooth method to widen the spatial resolution, bootstrap technique to estimate increased sample size, and a high resolution technique which is Multiple Signal Classification (MUSIC) to estimate DOA. Through different simulations, performance and effectiveness of the proposed approach, referred to as Spatial Smooth and Bootstrapped technique "SSBoot'', are demonstrated.**

Keywords—Direction of arrival (DOA) estimation; Bootstrap; Multiple Signal Classification (MUSIC); resolution; spatial smoothing; array processing; Uniform Linear Array (ULA)

I. INTRODUCTION

When two sources are very close in space in the ambiguity range, the radar detects them like one target. The spatial resolution limits for two closely spaced sources in the context of array processing is still an active research [1]-[3]. In fact, there has been a tremendous involvement in the investigation of how DOA estimation of many closed source (targets) can be estimated. Most of them, [4], [5], are based on high resolution methods, e.g. Multiple Signal Classification (MUSIC) or Estimation of Signal Parameter via Rotational Invariance Technique (ESPRIT), and detect sources using eigenvalues obtained from covariance matrix of samples. However, the main issue of high resolution method for DOA's estimation is predetermination of the model order, since these techniques requires imperatively number of sources, as input parameter within estimation. This estimation is based on information theoretic criteria like AIC (AKAIKE) and Rissanen's minimum description length criterion (MDL) algorithms to estimate DOA of sources [2], [5]-[7].

In other hand, performance of these techniques stays very poor for low samples, low SNR, correlated source signals and presence of impulsive white noise. To improve the resolution, a spatial smoothing technique is used. This technique divides the array into multiple overlapping sub-arrays. In each sub-array, the correlation matrix is estimated from bootstrapped data samples. We exploit the idea of the author in [3] and applied MUSIC in each sub-array; to estimate the number of sources as number of peaks [1], [8], [9].

Unfortunately, the most existing methods are less efficient and lost large performance or even breakdown when only few samples of received signal are available. To reduce this hurtful effect and improve the robustness of the covariance estimator, a robust non-parametric bootstrap method estimator was proposed [10]-[13]. Based on time random sampling of original data, to estimate its sampling distribution without any model assumption.

In this work, a new algorithm is proposed. It combines spatial smoothing, a high resolution method (MUSIC) and Bootstrap technique to estimate closely spaced number of sources and their DOA's when only few samples of received signal are available. First, it's used bootstrap method to estimate the covariance matrix, then spatial smoothing curves up the antenna array into L sub-networks. In each sub-network, MUSIC algorithm allows to estimate the number of closely spaced sources and their DOA's. Numerical simulations are given to assess the performance of the used technique.

The paper is organized as follows. Data, array model and MUSIC description are introduced in Section 2, followed by spatial sampling model description in Section 3. Then bootstrap technique is presented in Section 4. The proposed algorithm "SSBoot" is described in Section 5. Simulation results are given in Section 6. Finally, discussion and conclusion are given in Section 7.

II. PROBLEM FORMULATION

Just to simplify the notation, we assume a Uniform Linear Array (ULA) composed of M sensors, with equip-spacing $d=\lambda/2$ as shown in Fig. 1; where λ is the wavelength of the source signal. Consider a K narrowband far-field uncorrelated source impinging on the array with ($M > K$), such that sources have a direction of arrival (DOA) θ_k, with k=1... K.

A. Array Signal Model

The received snapshots at this array, at instance t are given by [1], [14].

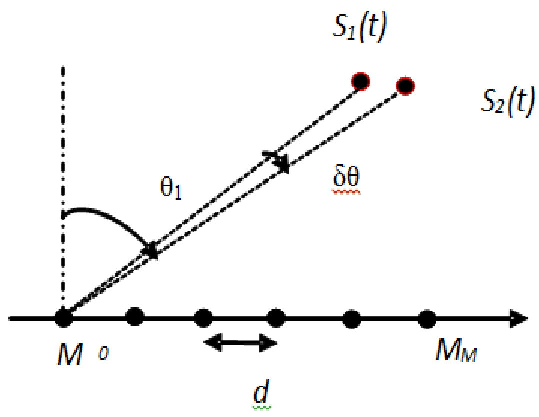

Fig. 1. Localization of two closely spaced sources impinging on ULA.

The received signal corrupted by additive white Gaussian noise is presented at instance t by mathematical equation [1], [15], [16]:

$$y(t) = A\, s(t) + n(t) \tag{1}$$

Where

$$A = [a_1 \dots a_{1k}]^T \tag{2}$$

is the steering matrix (MxK) full rank,

$$a_k = \left[1 \;\; a_k^1 a_k^2 \dots a_k^{(M-1)}\right]^T \tag{3}$$

and each column is written in function of the received signal as follows:

$$a_k = e^{(-j\, 2\pi (d/\lambda) \sin\theta_k)} \tag{4}$$

$$y(t) = [y_1(t) \;\dots\; y_M(t)]^T \tag{5}$$

$$S(t) = [S_1(t) \;\dots\; S_K(t)]^T \tag{6}$$

$$n(t) = [n_1(t) \;\dots\; n_M(t)]^T \tag{7}$$

Superscript $(.)^T$ presents the transpose operation. Where $y_k(t)$ denotes the output of k^{th} sensors, $s_q(t)$ source signal and $n_k(t)$ is a stationary noise model, temporally white, zero-mean Gaussian random process independent of the source signals. The covariance of received data is [1], [17], [18]:

$$R_{yy} = E\left[Y.Y^H\right] = AR_S A^H + \sigma^2 I \tag{8}$$

Where

$$R_S = E\left[S(t) S^H(t)\right] \tag{9}$$

The superscript $(.)^H$ stands for the conjugate transposition, σ^2 is variance and I indicate the identity matrix.

Furthermore, the covariance matrix is estimated by [2], [3], [17], [18]:

$$R_{yy} = \frac{1}{N} Y.Y^H \tag{10}$$

The eigenvalues are given as follows:

$$\rho_1 \geq \rho_2 \geq \dots \geq \rho_k \geq \rho_{k+1} = \rho_M = \sigma^2$$

where the first K eigenvalues belong to the source signal, and the last (M-K) to the noise.

MUSIC plots the pseudo-spectrum [2], [19]:

$$V_{Music} = \frac{1}{a^H(\theta) E_n E_n^H a(\theta)} \tag{11}$$

Where En is the (Mx(M-K)) noise subspace composed of the eigenvectors associated with the noise.

If we assume two closely spaced source where their DOA are θ_1 and θ_2 such as:

$$\theta_1 = \theta_2 + \delta\theta \quad \text{with } \delta\theta \prec 5°$$

B. Spatial Smoothing

In this section, it's described the use of spatial smoothing in proposed algorithm in order to improve resolution of very close spaced sources. The ordinary spatial smooth consists of dividing the whole array into L sub-arrays shifted one another by one sensor; the rest of sensors are overlapped as shown in Fig. 2. It estimates the correlation matrix as the average of all correlation matrices from the sub-arrays $\hat{R}^l$ and can be represented as:

$$\hat{R} = \frac{1}{L} \sum_{l=1}^{l=L} \hat{R}^l \tag{12}$$

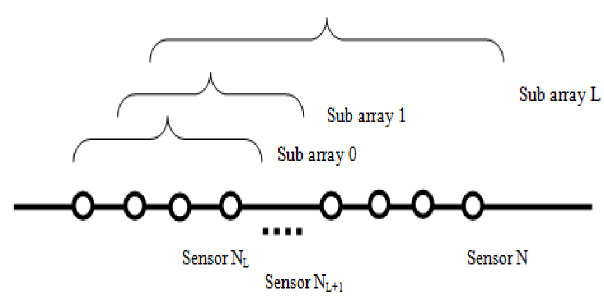

Fig. 2. An ULA antenna is divided into L sub array.

Our method is based on representations of Abed-Meraim et al. in [7] who divided the whole array into interleaving sub-arrays. In each sub-arrays the received signal is given by:

$$Y_m = \begin{bmatrix} y_1(0) & .... & y_1(T) \\ y_2(0) & .... & y_2(T) \\ . & & . \\ . & & . \\ y_{((N_L-1))}(0) & .... & y_{((N_L-1))}(T) \end{bmatrix} \quad (13)$$

Where m is ( )th array and varies from 1 to L, and N_L= M/L.

The same, the steering matrix for the m array is given by:

$$A_m(\theta) = \begin{bmatrix} 1 & .... & 1 \\ e^{j\pi \sin\theta_1} & .... & e^{j\pi \sin\theta_q} \\ e^{j\pi 2 \sin\theta_1} & .... & e^{j\pi 2\sin\theta_q} \\ . & & . \\ . & & . \\ e^{j\pi(N_L-1)\sin\theta_1} & .... & e^{j\pi(N_L-1)\sin\theta_q} \end{bmatrix} \quad (14)$$

Where T is a snapshot number and q is the number of source signal received in each sub-array.

In this case, and unlike results of Abed-Meraim in [3], [19], we are sure that number of sensors is always greater than number of sources, and therefore it respects the assumption to apply MUSIC.

Thus, that spatial smoothing or spatial sampling considerably improves the resolution. Indeed, according to (13) and (14), the angular part in the matrix output are multiplied by a factor (NL-1) which is greater than 1. Therefore, the angular separation is widened and the resolution is improved.

C. *Bootstrap Replication*

In this section, non-parametric bootstrap resampling techniques are presented, designed for independent and identically distributed data. However, the assumption of iid data can break down during operation either because data are not independent or because data are not identically distributed [7], [8].The original data points:

$$\mathrm{x} = (\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3 \dots . \mathrm{x}_\mathrm{n}) \quad (15)$$

with probability $\frac{1}{\mathrm{n}}$ for each sample.

A bootstrap sample X* is obtained through replacement of original data points by random sampling (n times) [10]-[12].

Some bootstrap samples can be:

$$\mathbf{x}^{(1)} = (\mathrm{x}_2, \mathrm{x}_5, \mathrm{x}_\mathrm{n} \dots . \mathrm{x}_1)$$
$$\mathbf{x}^{(2)} = (\mathrm{x}_1, \mathrm{x}_4, \mathrm{x}_1 \dots . \mathrm{x}_\mathrm{n})$$
$$\mathbf{x}^{(\mathrm{B})} = (\mathrm{x}_1, \mathrm{x}_4, \mathrm{x}_1 \dots . \mathrm{x}_\mathrm{n}) \quad (16)$$

with n samples.

We assume that the x_i's are independent identically distributed (iid), each having distribution F. Bootstrap proposes to resample from a distribution chosen to be close to F in some sense. This could the empirical distribution$\hat{F}$, resampling from $\hat{F}$is referred to as non-parametric bootstrap [10].

At the end we obtain:

$$\boldsymbol{X}^* = (x_1^*, x_2^*, x_3^*, \dots . x_n^*) \quad (17)$$

Herein, we create a number B of resamples $\mathrm{X}_1^*, \dots, \mathrm{X}_\mathrm{B}^*$. The resampled boostrapis an unordered collection of n samples points drawn randomly from X with replacement, so that each X_i^*has probability $\frac{1}{\mathrm{n}}$of being equal to any one of the X_j' s. In other terms [8], [9], [11]:

$$\mathrm{Prob}\left[X_i^* = X_j / X\right] = n^{-1}, \ 1 \le i, \ j \le n \quad (18)$$

This means that $\boldsymbol{X}^*$ is likely to contain repeats. The probability that a particular value x_i is left out is

$$P = (1 - \tfrac{1}{n})^n \quad (19)$$

We exploits the resample bootstrap algorithm described in [9] to reproduce samples and use it in our proposed "SSBoot" algorithm.

III. Spatial Smooth Bootstrapped "Ssboot" Algorithm

Firstly, the proposed method is based on increasing the number of snapshots received on array network using bootstrap technique. Secondly, each of sub-arrays is processed separately and finally the average DOA estimation is considered.

Determination number of sources first is essential for high-resolution method. It should use AIC or MDL algorithm to determine the model order. But, in this work, we followed the same spirit given in [3], [7]. We estimated the source number using beamforming or Capon method applied to the global array output. If q peaks appear, we re-apply MUSIC algorithm by restricting our research in intervals around each q peaks.

Applying spatial smoothing yields to divide the array network into L overlapping sub-arrays thus, we obtain L different DOA's estimates. Among these L sets, we keep only the highest number of peaks in each interval.

Our new algorithm, we named "SSBoot" can be summarized as follows:

- *Step 1*: Applying bootstrap technique to generate new samples by sampling with replacement of original data.
- *Step 2*: First estimation number of sources on global array network using Capon method.
- *Step 3*: Defining set of intervals where search are refined.
- *Step 4*: Divide the global antenna array into L shifted overlapped sub-arrays.
- *Step 5*: On each sub-array, we apply MUSIC Algorithm. The number of MUSIC spectrum peaks equals to number of sources.

- *Step 6*: The number of sources is selected from p intervals for L sub-arrays that present maximum number peaks,
- *Step 7*: Computing the final DOA, after sorting and calculating the average from each interval and selecting sub-arrays with maximum peaks.

$$\widehat{\theta_j} = \frac{1}{p}\sum_{l=1}^{p} \widehat{\theta_j^l} \quad (20)$$

Where $\widehat{\theta_j^l}$, l =1..p represents p estimates DOA from different sub-arrays.

IV. Simulation and Result

To illustrate the performance of the proposed method, some numerical results are presented to analyse and compare behaviour estimation of the new proposed algorithm which is named "SSBoot". A Uniform Linear Array (ULA) is constituted of N=10 sensors spaced of half-length wave length is employed. Assume that there are two closely spaced uncorrelated narrowband signal sources with the same wavelength λ, θ_1= 32° and θ_2= θ_1+δθ, where δθ is a very small angle difference. Simulation results were obtained based on 1000 Monte Carlo simulation.

Performance of bootstrap for varying snapshots for arrival angles of -40° -20° 60° and 80° respectively, are illustrated in Fig. 3. When a few samples (20 snapshots) are received the MUSIC spectrum response is almost flat and the DOA is difficult to extract, but when these samples are bootstrapped at 200, 1000 the 2000 samples, the responses increases and the peaks become noticeable. However, it demonstrates the effectiveness of the bootstrap method to improve the detection and estimation of DOA.

Fig. 4 presents the probability of target detection in percentage for various angular separations; it illustrates the performance achieved by our method for few snapshots with low SNR. In fact, for received low samples, the detection is weak; it increases slowly when SNR increases. But when these samples are bootstrapped at 1000 snapshots the estimation rate improves and reaches the maximum rate with low SNR. However, our algorithm SSBoot bootstraps the received samples and uses the spatial sampling to improve its estimation performance for the same number of snapshots. Indeed the very close spaced sources are detected for low SNR.

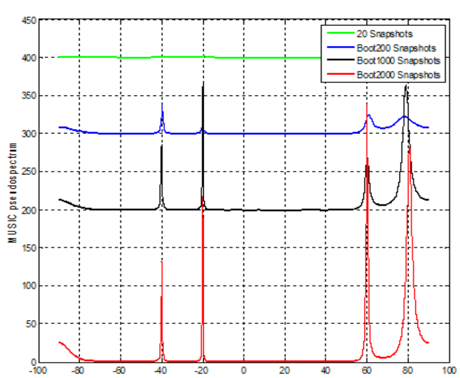

Fig. 3. MUSIC Spectrum for various snapshots.

Fig. 5 depicts the probability of detection rate in percentage for various SND in dB; it shows that for few samples the detection nearly breaks down. With bootstrap at 1000 snapshots, the detection is slightly achieved because of low SNR values. The SSBoot proposed method overcame this limitation by ensuring a highest detection rate for low SNR and very close separation sources.

Fig. 6 illustrates, the DOA's MSE (Mean Square Error) vs. SNR for L=2, and angle difference δθ =5°, it can be observed that the MSE for Only bootstrapped MUSIC method and our technique SSBoot that uses Bootstrap, spatial smooth and MUSIC have almost the same estimation accuracy. It means that SSBoot improves the resolution with no estimation accuracy enhancement.

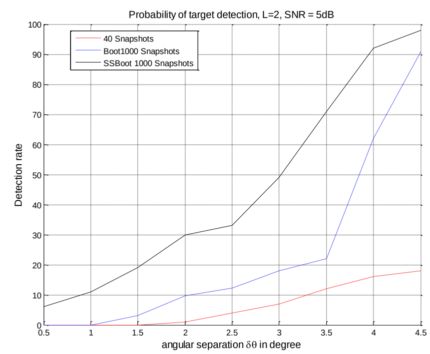

Fig. 4. Angular separation vs. detection rate.

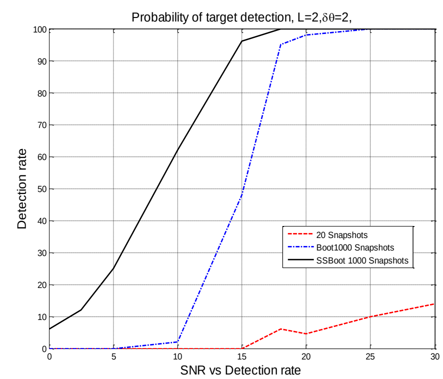

Fig. 5. SNR vs. Detection rate.

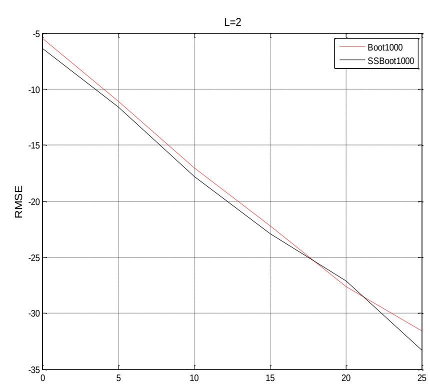

Fig. 6. RMSE vs. SNR.

V. CONCLUSION

In this paper, we have introduced a new technique based on the combination of bootstrap technique, spatial smoothing and MUSIC method to improve resolution and the estimation of closed source number. It was shown that for the case of small sample size, the bootstrap technique is used to estimate and evaluate the resample data. The spatial smoothing was also presented as spatial sampling method, which provides different sub-arrays and widens the angle separation of closed source when MUSIC Algorithm is applied.

The results presented in this paper prove that our method is attractive when few samples are available and outperforms the ordinary technique at difficult scenarios especially for very close source and low SNR. Simulations have shown that spatial sampling and bootstrap techniques outperforms DOA estimation, when MUSIC method is applied for small sample size and very close sources. But it’s demonstrated that SSBoot technique can’t improve the estimation accuracy.

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