Fuzzy Pi Adaptive Learning Controller for Controlling the Angle of Attack of an Aircraft

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Abstract—In this paper, a Fuzzy PI Adaptive Learning controller is proposed for a flight control system to control the angle of attack of an aircraft. The proposed controller tracks the reference angle as desired by the pilot of the aircraft. The performance indices are evaluated and the corresponding value is compared with that for the conventional controllers obtained from Zigler Nichols (ZN), Tyreus Luyben (TL) and Extended Skogestad Internal Model Controller (ESIMC). The performance indices such as Mean Square Error (MSE), Integral Absolute Error (IAE) and Integral Absolute Time Error (IATE) are evaluated to verify superiority of one over another.

Keywords—Angle of Attack; Interpolation Rule; Performance Indices; Fuzzy PI Adaptive Learning Controller

I. INTRODUCTION

An aircraft flies in a 3D space controlled by its control surfaces such as aileron, rudder and elevator. Generally the motion of aircraft is changed by these control surfaces, but the angle of attack of the aircraft is controlled by the deflection of the elevator. Since to control the angle of attack of an aircraft is very crucial, therefore fuzzy controllers are frequently used to offer better and accurate output as compared to conventional controllers ZN, TL and ESIMC (Interpolation Rule).

In 1958, W. Gracey [1] summarised about the methods of measuring the angle of attack of an aircraft in a precise manner. C. Grimholt [2] gives an idea of improving the Skogestad Internal Model PI control strategy. M Shamsuzzoha and S. Skogestad [3] discussed about the set-point overshoot method for a closed loop PID controller. S Yordanova and E Haralanova [4] designed and implemented a robust multivariable PI fuzzy controller for an aerodynamic system. F. Dimeas and N. Aspragathos [6] proposed a Fuzzy Learning Variable Admittance Control for a Human Robot system. I.S Baruch and S. Hernandez [7] discussed about a decentralised direct I-term Fuzzy-neural controller for controlling an anaerobic digestion bioprocess system. Lian Ruey-Jing [8] proposed an adaptive self-organising fuzzy sliding mode, Radial Basis Function Neural Network controller for robotic mechanism. S. Kamalasadan and A.A Ghandakly [9] proposed Neural Network based parallel adaptive controller to track the pitch rate of a fighter aircraft. Huang Huazhang and Chung Chi-Yung [10] implemented an adaptive neuro fuzzy Controller for static VAR compensator to damp out the oscillations of wind energy. Guo Lusu and L Parsa [11] designed a Model reference adaptive controller for a five

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phase IP motor. Dawood Amoozegar [12] proposed about the modelling of a DSTATCOM for stability analysis of the voltage with the help of a fuzzy logic PI current controller. K. Premkumar and B.V. Manikandan [13] designed an Adaptive neuro-fuzzy inference system to control the speed of a brushless DC motor. E.A. Ramadan, M. El-bardini and M.A. Fkirin [14] designed and implemented FPGA to control the speed of a DC motor using an adaptive fuzzy controller. A. Fereidouni, M.A.S. Masoum and M. Moghbel [15] proposed a new adaptive fuzzy PID controller. J. Yoneyama [16] designed a nonlinear control system based on generalised Takagi-Sugeno fuzzy systems.

In this paper, an adaptive fuzzy PI controller is implemented for controlling the angle of attack of an aircraft. Then the performance indices (MSE, ISE & ITAE) of the aircraft are evaluated and the results are compared with the conventional Zeigler Nichols, Tyreus Luyben and Skogestad Internal Model Control techniques and it was established that the adaptive fuzzy PI controller gives excellent results which improves the performance indices and reduces the error.

II. ANGLE OF ATTACK CONTROL SYSTEM

Figure 1 below depicts the block diagram representation of the angle of attack with disturbance and controller. In this diagram input is the elevator deflection and output is the angle of attack.



Fig. 1. Angle of attack with disturbance and controller

- G(s) = Transfer function of angle of attack
- C(s) = Fuzzy PI controller Transfer Function
- $G(s) = G_1(s) =$ Disturbance where,
- $\delta_E(s) =$ The elevator deflection
- α = The angle of attack

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Angle of attack is defined as the angle between the chord line of the wing and the relative motion between aircraft and atmosphere. It is controlled by the elevator deflection. Figure 2 below illustrates the angle of attack and the direction of relative wind.



Fig. 2. Angle of attack and the direction of relative wind

Considering the short period approximation (speed of the aircraft u=constant) the longitudinal dynamics [5] of the aircraft reduces to elevator deflection, then using vector matrix notation, Equation (1) and Equation (2) may be written as

$$\dot{w} = Z_w w + U_0 q + Z_{\delta_E} \delta_E \tag{1}$$

$$q = M_{\dot{w}}w + M_{w}\dot{w} + M_{q}q + M_{\delta_{E}}\delta_{E} = (M_{\dot{w}} + M_{w}Z_{w})w$$
⁽²⁾

$$+ \left(M_q + U_0 M_w\right) q + \left(M_{\delta_E} + Z_{\delta_E} M_{\dot{w}}\right) \delta_E \tag{2}$$

If $x = \begin{bmatrix} w \\ q \end{bmatrix}$ = the state vector and u= the control vector =

$$\dot{x} = Ax + Bu \tag{3}$$

where,

$$A = \begin{bmatrix} Z_{w} & U_{0} \\ (M_{w} + M_{w}Z_{w}) & (M_{q} + U_{0}M_{w}) \end{bmatrix}$$
$$B = \begin{bmatrix} Z_{\delta_{E}} \\ M_{\delta_{E}} + Z_{\delta_{E}}M_{w} \end{bmatrix}$$

Now

$$\begin{bmatrix} sI - A \end{bmatrix} = s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} Z_{w} & U_{0} \\ (M_{w} + M_{\dot{w}}Z_{w}) & (M_{q} + U_{0}M_{\dot{w}}) \end{bmatrix}$$
$$= \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} Z_{w} & U_{0} \\ (M_{w} + M_{\dot{w}}Z_{w}) & (M_{q} + U_{0}M_{\dot{w}}) \end{bmatrix}$$
$$= \begin{bmatrix} s - Z_{w} & -U_{0} \\ -(M_{w} + M_{\dot{w}}Z_{w}) & [s - (M_{q} + U_{0}M_{\dot{w}})] \end{bmatrix}$$
(4)

Again,
$$\Delta_{sp}(s) = \det[sI - A] = s^{2} + \left[-\left(Z_{w} + M_{q} + U_{0}M_{w}\right) \right]s + \left[Z_{w}M_{q} - U_{0}M_{w}\right]$$
$$= s^{2} + 2\zeta_{sp}\omega_{sp}s + \omega_{sp}^{2}$$
(5)

The transfer function is given by

$$\frac{w(s)}{\delta_E(s)} = \frac{\left(U_0 M_{\delta_E} + M_q Z_{\delta_E}\right) \left\{1 + \frac{Z_{\delta_E}}{U_0 M_{\delta_E} - M_q Z_{\delta_E}} s\right\}}{\Delta_{sp}(s)} = \frac{K_w(1 + sT_1)}{\Delta_{sp}(s)}$$
where,
$$(6)$$

 $K_{w} = \left(U_{0}M_{\delta_{E}} + M_{q}Z_{\delta_{E}}\right)T_{1} = \frac{Z_{\delta_{E}}}{K_{w}} \cdot$

Again,

$$\dot{\alpha} = \frac{\dot{w}}{U_0} \Longrightarrow \alpha(s) = \frac{w(s)}{U_0} \Longrightarrow w(s) = U_0 \alpha(s) \tag{7}$$

From Equation (6) and Equation (7), the transfer functions for angle of attack is given by

$$\frac{\partial \alpha(s)}{\partial \delta_{E}(s)} = \frac{\left(U_{0}M_{\delta_{E}} + M_{q}Z_{\delta_{E}}\right)\left\{1 + \frac{Z_{\delta_{E}}}{U_{0}M_{\delta_{E}} - M_{q}Z_{\delta_{E}}}s\right\}}{U_{0}\Delta_{sp}(s)} = \frac{K_{w}(1 + sT_{1})}{U_{0}\Delta_{sp}(s)}$$

$$\Rightarrow \frac{\alpha(s)}{\delta_{E}(s)} = \frac{K_{w}(1 + sT_{1})}{U_{0}\Delta_{sp}(s)}$$
(8)

The above values are the stability derivatives [5] of longitudinal dynamics of FOXTROT aircraft as shown in "Table 1" below.

TABLE I. STABILITY DERIVATIVES OF FOXTROT AIRCRAF	TABLE I.	STABILITY DERIVATIVES OF FOXTROT AIRCRAFT
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Stability	Flight Condition(FC)				
Derivatives	FC-1	FC-2			
$U_0\left(ms^{-1}\right)$	70	265			
Z_u	-0.117	-0.088			
Z_w	-0.452	-0.547			
Z_q	-0.76	-0.88			
M_{u}	0.0024	-0.008			
$M_{_W}$	-0.006	-0.03			
$M_{\dot{w}}$	-0.002	-0.001			
M_q	-0.317	-0.487			
X_{δ_E}	1.83	0.69			
$\overline{Z_{\delta_E}}$	-2.03	-15.12			
M_{δ_E}	-1.46	-11.14			

The transfer function for both the flight conditions (Flight Condition-1 and Flight Codition-2) are obtained after substituting the values of the stability derivatives mentioned in "Table 1" above. Now the transfer functions are given by

Flight Condition-1

$G(s) = \frac{2.0302s + 102.8}{1000000000000000000000000000000000000$	3.604s + 182.5
$O_1(3) = \frac{1}{s^2 + 0.901s + 0.5633}$	$-\frac{1.7752s^2+1.5798s+1}{1.7752s^2+1.5798s+1}$
Flight Condition-2	
C(s) = 15.11s + 0.003027	1.84s + 368.5
$U_{2}(3) =$	

$$s^{2}(3) = s^{2} + 1.2989s + 8.216 = 0.1217s^{2} + 0.1581s + 1$$

III. CONVENTIONAL PI CONTROLLERS

The transfer function for PI controller [C(s)] is given by $C(s)=K_p+(K_i/s)$ and the values of K_p and K_i are determined by various types of conventional PI controllers, such as Zeigler Nichols, Tyreus Luyben and Extended Skogestad Internal Model Controller and are discussed as follows:

A. Zeigler-Nichols (ZN) PI controller

In this method, the PI controller [2] parameters K_p and T_i depends on the value of ultimate gain K_u and ultimate period

 $P_{\rm u}$ for sustained oscillations. The value of PI controller parameters is shown in Table 2 below.

TABLE II. VALUES OF K_P And T_I for 2N Controller

PID	K	T
Туре	κ _p	1 _i
PI	$\frac{k_u}{3.2}$	2.2 <i>P</i> _u

B. Tyreus-Luyben(TL) PI Controller

In this type of controller [3], the oscillations are minor and the controller is robust unlike Zeigler Nichols and the tuning parameters K_p , and T_i are illustrated in the Table 3 below.

TABLE III. VALUES OF K_P and T_I for TL Controller

PID Type	K_p	T_i
PI	$0.45k_{u}$	$\frac{P_u}{1.2}$

C. Extended Skogestad Internal Model (ESIMC) PI Controller (Interpolation Rule)

In this type of controller [2], the values of K_p and K_i for proportional and integral controller are given by

 $K_{p} = \max\{A, X\},\$

where,
$$X = B$$
 for $\zeta \ge 1$ and $X = \zeta B' + (1 - \zeta)C$ for $\zeta < 1$

 $K_i = \max\left\{A, X\right\},\,$

where, X = B for $\zeta \ge 1$ and $X = \zeta B' + (1 - \zeta)C$ for $\zeta < 1$

The values of A, B, B' and C for proportional and integral controllers are given in Table 4 below.

TABLE IV. THE VALUES OF A, B, B' AND C

	Calculation of $K_{_C}$	Calculation of K_I
Α	25	1
	$k \left(\tau_{c} + \theta \right)$	$\tau_0 k \left(\tau_c + \theta\right) \tau_0^2$
В	$\frac{1+4(\tau_c+\theta)+\frac{(\zeta+\sqrt{\zeta^2})}{\tau_0}}{\frac{\tau_0}{\tau_0}}$	$\frac{-1}{k^{"}\left(\zeta+\sqrt{\zeta^{2}-1}\right)} \frac{k^{"}\left(\tau_{c}+\theta\right)^{2}\tau_{0}}{k^{"}\left(\tau_{c}+\theta\right)^{2}\tau_{0}}$
	$\kappa (\iota_c + 0)$	<i>a a</i>
В	$\frac{1+4(\tau_c+\theta)+\frac{1}{2}}{2}$	$\frac{\zeta}{k''(\tau_c + \theta)\tau_0}$
	$k^{"}(\tau_{c}+\theta)^{2}$	
С	1	1
	$\overline{2k^{"}(au_{c}+ heta)^{2}}$	$\overline{16k^{"}(\tau_{c}+\theta)^{3}}$

In Table 4 above,

$$k'' = \frac{k}{\tau_0^2}$$

k = The gain

$$\tau_0 = \frac{1}{\omega_n}$$

 ω_n = Natural frequency of oscillation

 $\zeta =$ Damping ratio

 τ_c = The controller tuning parameter

$$\theta = \tau_0 (1.5 + 0.5\zeta)(0.6)^a$$
 = the delay angle

$$a = \tau_0^2$$

B is obtained by setting $\sqrt{\zeta - 1} = 0$ in *B*.

D. Result Analysis for Conventional Controllers

The simulations for above three controllers are done by the help of Matlab 7.1. The step response of controller output 'u' and the system output (angle of attack) 'y' for three controllers for Flight Condition-1 and Flight Condition-2 with set-point and disturbances are shown in Figures 3 to 6, respectively.



Fig. 3. Step response of 'u' with set-point and disturbance for flight condition-1



Fig. 4. Step response of 'y' with set-point and disturbance for flight condition-1



Fig. 5. Step response of 'u' with set-point and disturbance for flight condition-2



Fig. 6. Step response of 'y' with set-point and disturbance for flight condition-2

IV. ADAPTIVE FUZZY LEARNING CONTROLLER (AFLC)

An adaptive Fuzzy PI Controller [6] utilises a learning mechanism for controlling the angle of attack and adjusts the rule base such that the overall system behaves like a reference model. The fuzzy controller improves the stability of a timevariant non-linear system by tuning controller parameters. Figure 7 below shows functional block diagram of the controller.



Fig. 7. Functional Block Diagram of Fuzzy Learning Controller

A. Fuzzy Rule Base

It is nothing but a set of if-then rules according to which the Fuzzy Controller operates to control the angle of attack of an aircraft. The rule base for the present work is shown in Table 5 below.

FABLE V. R	ULE BASE FOR THE ANGLE OF ATTACK FUZZY MODEL

Elevat	Change in Error											
Deflection		-5	-4	-3	-2	-1	0	1	2	3	4	5
	-5	5	5	5	5	5	5	4	3	2	1	0
	-4	5	5	5	5	5	4	3	2	1	0	-1
	-3	5	5	5	5	4	3	2	1	0	-1	-2
Error	-2	5	5	5	4	3	2	1	0	-1	-2	-3
	-1	5	5	4	3	2	1	0	-1	-2	-3	-4
	0	5	4	3	2	1	0	-1	-2	-3	-4	-5
	1	4	3	2	1	0	-1	-2	-3	-4	-5	-5
-	2	3	2	1	0	-1	-2	-3	-4	-5	-5	-5
	3	2	1	0	-1	-2	-3	-4	-5	-5	-5	-5
	4	1	0	-1	-2	-3	-4	-5	-5	-5	-5	-5
	5	0	-1	-2	-3	-4	-5	-5	-5	-5	-5	-5

B. Fuzzy Membership Functions

The membership functions characterise the situations for application of the fuzzy rules. In this work the membership functions for input and output are taken into consideration. The membership functions input universe of discourse is assumed to be constant and are not tuned by adaptive controller whereas that for output universe of course are known.

In this work the tuning parameters $g_e = 2/\pi$, $g_c = 250$ and $g_u = 8\pi/18$ for an output universe of discourse [-1, 1] are triangular in shape with base widths of $0.4*g_u$ and centres at zero are chosen. This choice represents that the fuzzy controller initially knows nothing about how to control the plant so it inputs u = 0 to the plant initially. Fuzzy controller input and output membership functions are depicted in following Figures 8 and 9, respectively.



Fig. 8. Fuzzy controller input Membership Function



Fig. 9. Fuzzy controller output Membership Function

C. The Learning Mechanism

The rule base of the fuzzy controller is tuned by the learning mechanism to make the close loop system a reference model. The modification of rule base is done according to the output of controller and the reference model. The learning mechanism is divided into two parts. The first part is the fuzzy inverse model and the second part is the rule base modifier. The fuzzy inverse model maps with the change in input required to force the output to zero. In this paper, membership functions for the input universes of discourse are symmetrical triangular-shaped.

D. Rule Base Modifier

The rule base of the fuzzy controller can be changed by rule base modifier to force the error of the control action to zero. The input to the fuzzy controller is the error signal and the change in error signal. The rule base can be changed by shifting the centres of the membership functions as depicted in Figure 10 below.



Fig. 10. Shifting of Centers of Membership Functions

E. Simulation Results of the Adaptive Fuzzy Learning Controller (AFLC)

The simulation is done by using Matlab 7.1. The simulation is done by taking two cases into consideration.

- 1) Case-I: Simulation without Sensor Noise
- 2) Case-II: Simulation with Sensor Noise

1) Case-I: Simulation without Sensor Noise: In this case the reference signal is applied for a duration of 40 seconds out of which the first 25 seconds is for FC-1 with a speed of 70m/dssec and the next 15 seconds for FC-2 with a speed of 265m/sec. Initially, AFLC has no adaptation but as the flight proceeds the controller gets adapted with changing the centre of membership function.

Figure 11-a depicts the angle of attack and desired angle of attack whereas Figure 11-b shows the elevator deflection i.e. input to the aircraft which is output from the fizzy controller. Similarly, Figure 11-c depicts the Fuzzy inverse model output in which the non-zero values indicates the adaptation. Again, Figure 11-d depicts the error between the actual and desired values whereas Figure 11-e depicts the change in error. Figure 11-f shows the error between angle of attack and the reference model and Figure 11-g shows the corresponding change in error.



Fig. 11. Responses without Sensor Noise

Time (sec)



Fig. 12-a: Angle of attack and desired Angle of attack

Fig. 12-d: Error between Angle of attack and desired Angle of attack



Fig.12-f: Error between Angle of attack and reference model, deg.



Fig. 12. Responses with Sensor Noise

2) Case-II: Simulation with Sensor Noise: In this case the pulse duration is also 40 seconds for the reference model. A random noise $0.01 \frac{\pi}{2} (2*rand-1)$ is added uniformly with the Angle of attack to¹% rify the adaptive nature of the controller. Figure 12 depicts the results of the simulation of all the parameters of Figure 11 in presence of the noise and it is clear that controller is noise adaptive.

F. Control Surface

Figures 13 and 14 shows the control surfaces [5] of AFLC without and with sensor noise, respectively. It reveals from figure that the control surface is non-linear in nature. This non-linearity nature of control surface changes with change in system parameters and is indicated by the angle of attack error and change in angle of attack error.



Fig. 13. Control Surface Without Sensor Noise



Fig. 14. Control Surface With Sensor Noise

G. Performance Indices

The performance indices of the system are given by

$$IAE = \int_{0}^{\infty} |e(t)| dt, MSE = \frac{1}{T} \int_{0}^{\infty} e^{2}(t) dt, IATE = \int_{0}^{\infty} t |e(t)| dt$$

where, the control error, $e = \alpha - \delta_{E}$

The performance indices of Zeigler Nichols Controller, Tyreus Luyben Controller, Extended Skogestad Internal Model Controller and Adaptive Fuzzy Learning Controller are compared to establish the superiority of adaptive fuzzy controller over other three controllers. It was also established that AFLC gives better results as depicted in Table 6 below.

Controllors	Performance Indices							
Controllers	MSE	IAE	IATE					
ZN	0.1311	57.3971	63.0637					
TL	0.1256	53.4471	30.6712					
ESIMC	0.0973	27.3914	1.9471					
AFLC	0.0698	19.3787	1.1146					

TABLE VI. PERFORMANCE INDICES OF ZN, TL, ESIMC AND AFLC

V. CONCLUSION

In this paper, the angle of attack of the aircraft is controlled using various techniques and the results are depicted in Figures 3, 4, 5, 6, 11 and 12. Also the performance indices of the system are compared as shown in Table 5 above. It reveals that AFLC adapts the change in flight conditions from FC-1 to FC-2 and gives excellent results, improves the performance indices and reduces the errors. The performance indices MSE, IAE and IATE are very less as compared to ZN, TL and ESIMC controllers. The proposed controller not only tracks the desired angle of attack but also noise adaptation. In case of noisy input (Figure 12-b) the nonzero values of the controller output indicates that the controller continuously sends the output which nullifies the error to track the desired angle of attack. Therefore, AFLC can also be applied to other dynamic systems for its better performance and output.

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