

# Non-Linear Energy Harvesting Dual-hop DF Relaying System Over $\eta - \mu$ Fading Channels

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**Abstract**—In this work, we analyze a wireless energy harvesting decode-and-forward (DF) relaying network with beamforming that is based on a practical non-linear energy harvesting model over  $\eta - \mu$  fading channels. We consider a dual-hop relaying system having multiple antennas at the source and destination only. The single-antenna energy constrained relay assists the source to communicate with the destination. At the relay node, we assume a non-linear energy harvesting receiver which limits the harvested power level with a saturation threshold. By considering a power-splitting based relaying (PSR) protocol and a non-linear energy harvesting receiver, we analyze the system performance in terms of the outage probability and throughput for various antennas combinations and for various values of the fading parameters,  $\eta$  and  $\mu$ . The  $\eta - \mu$  fading model has a few particular cases, viz., Rayleigh, Nakagami- $m$ , and Hoyt. These results are general and can be reduced for different fading scenarios as well as for linear energy harvesting relaying.

**Keywords**—Energy harvesting relay; non-linear energy harvester;  $\eta - \mu$  fading; power-splitting-based relaying; throughput

## I. INTRODUCTION

Wireless energy harvesting is a method by which energy is harvested from radio frequency (RF) signals. It is a gorgeous solution to increase the wireless network lifetime [1] and got much interest in the last decade [1]–[13]. The RF signals can accommodate energy and information simultaneously, hence energy can be harvested from the RF signals and can be stored for rechargeable devices to operate [1].

Integrating a dual-hop relaying system with wireless energy harvesting techniques can enhance the coverage capacity and network life-time [2], [3]. In a dual-hop wireless energy harvesting relaying network, the relay is an energy constrained device which harvests the energy through RF signals [4]. One of the well-known relaying methods is a decode-and-forward (DF) relaying, in which relay decodes and sends the information to the destination [2]–[4]. Several authors have investigated the wireless energy harvesting in a DF relaying system [2]–[5], [7]–[13] (and references therein).

In the works [2]–[4] (and their references), for the dual-hop energy harvesting DF relaying networks, a linear energy harvesting receiver was assumed. But, in reality, an energy harvesting circuit is non-linear owing to the non-linearity of the electronic components, i.e., inductors, capacitors, and diodes [7]. Therefore, a non-linear energy harvesting receiver is not a practical node and restrict the level of the harvested power. The performance of DF energy harvesting relaying systems with a non-linear energy harvesting receiver was studied over

conventional fading channels [7]–[13] and general  $\kappa - \mu$  shadowed fading channels [7].

Despite the importance of a non-linear energy harvesting receiver in a dual-hop energy harvesting relaying system, impact of the non-linear mode of energy harvesting receiver over  $\eta - \mu$  fading channels is not investigated so far.

Therefore, in this work, the effects of the non-linearity of the energy harvesting receiver and transmit/receive beamforming on the system performance of the considered system in  $\eta - \mu$  fading environments is studied. A PSR protocol [4] is assumed and the performance is evaluated for  $\eta - \mu$  fading channels. First, in a delay-limited transmission mode, the analytical results for the performance metrics (i.e., outage probability and throughput) are described. Then, the performance in the  $\eta - \mu$  fading scenarios is evaluated for different conditions. The  $\eta - \mu$  fading model is a generalized fading model and has a few particular cases, viz. Rayleigh, Nakagami- $m$ , and Hoyt [14]. Therefore, the general  $\eta - \mu$  fading scenario can be figured to some symmetric and asymmetric cases, namely, the Rayleigh/Rayleigh, Hoyt/Hoyt, Nakagami- $m$ /Nakagami- $m$ , and mixture of these fading links.

We sectioned our paper as: Section 2 explains the system and channel models; in Section 3, performance metrics are described; the distinctive cases are debated in Section 4; the simulated results are presented in Section 5; Section 6 summarizes the paper.

## II. SYSTEM AND CHANNEL MODELS

### A. System model

Consider a half-duplex dual-hop DF relaying system with energy harvesting where a source having multiple-antennas ( $N_1$ ) is transmitting the information to the destination having multiple antennas ( $N_2$ ) via a single-antenna relay. The relay is an energy constrained device which harvests the energy from RF signals of the source. The source and destination have no direct link, therefore, relay uses the harvested power to forward the source information to the destination. A PSR protocol [4] is assumed at the relay and transmission block  $T$  is splitted into two equal parts. During  $1/T$ , the relay divides the received RF signals into two portions with power splitting ratio  $\rho$ , where  $\rho \in (0, 1)$ , one for signal detection and the second for energy harvesting. In the second time slot  $T/2$ , employing the harvested power, the relay forwards the information signals to the destination. The destination unify all signals with MRC (maximum-ratio transmission) technique. Let  $\mathbf{h}_1$  and  $\mathbf{h}_2$  are

the  $N_1 \times 1$  and  $1 \times N_1$  channel vectors of the source-relay and relay-destination, respectively.

The received signal at the relay node can be written as [7]

$$y_R = \sqrt{P_s} \mathbf{h}_1^\dagger \mathbf{w}_1 x + n_{a,r} \quad (1)$$

where  $\mathbf{w}_1 = \frac{\mathbf{h}_1}{\|\mathbf{h}_1\|}$  [7], and  $P_s$  and  $x$  are the source transmit power and the normalized source information signal, respectively. Further, at the relay antenna,  $n_{a,r} \sim \mathcal{CN}(0, \sigma_{a,r}^2)$  is the additive white Gaussian noise (AWGN).

The received signal at the relay  $y_R$  is divided into two portions: Energy harvester utilizes the one of the portion  $\sqrt{\rho} y_R$  and the information receiver utilizes the other remaining portion  $\sqrt{1-\rho} y_R$ . During  $1/T$ , energy is collected by the energy harvester with an energy conversion efficiency  $\alpha$  as [7]

$$E_h = \alpha \rho P_s \|\mathbf{h}_1\|^2 \left(\frac{T}{2}\right). \quad (2)$$

A non-linear energy harvesting receiver is considered that emits a constant transmit power  $\alpha P_{th}$  when the input power is beyond a saturation threshold power  $P_{th}$  [7]. Therefore, the relay transmit power  $P_r$  can be [7]:

$$P_r = \frac{E_H}{T/2} = \alpha \rho \min\left(P_s \|\mathbf{h}_1\|^2, P_{th}\right) \quad (3)$$

$$= \begin{cases} \alpha \rho P_s \|\mathbf{h}_1\|^2, & P_s \|\mathbf{h}_1\|^2 \leq P_{th}, \\ \alpha \rho P_{th}, & P_s \|\mathbf{h}_1\|^2 > P_{th}. \end{cases} \quad (4)$$

It is clear from (3) that if  $P_s \|\mathbf{h}_1\|^2 \leq P_{th}$  then the energy harvesting receiver operates in a linear mode otherwise it operates as a non-linear device.

At the information processing receiver, the second portion of the signal  $\sqrt{(1-\rho)} y_R$  is given as

$$\sqrt{(1-\rho)} y_R = \sqrt{(1-\rho)} P_s \mathbf{h}_1^\dagger \mathbf{w}_1 x + \sqrt{(1-\rho)} n_{a,r} + n_{c,r}. \quad (5)$$

Using (5), the signal-to-noise ratio (SNR) at the relay terminal can be:

$$\gamma_R = \frac{(1-\rho) P_s \|\mathbf{h}_1\|^2}{(1-\rho) \sigma_{a,r}^2 + \sigma_{c,r}^2} \quad (6)$$

where  $n_{c,r} \sim \mathcal{CN}(0, \sigma_{c,r}^2)$  symbolizes the AWGN at the relay owing to RF-to-baseband transformation. Employing the harvested power,  $P_r$ , the relay sends the information to the destination is written by

$$\mathbf{y}_D = \sqrt{P_r} \mathbf{h}_2 \mathbf{w}_2 x_r + \mathbf{n}_{a,d} + \mathbf{n}_{c,d} \quad (7)$$

where  $\mathbf{w}_2 = \frac{\mathbf{h}_2}{\|\mathbf{h}_2\|}$  [7],  $x_r$  denotes the transmitted signal from the relay, and  $\mathbf{n}_{a,d} \sim \mathcal{CN}(0, \sigma_{a,d}^2 \mathbf{I}_{N_2})$  and  $\mathbf{n}_{c,d} \sim \mathcal{CN}(0, \sigma_{c,d}^2 \mathbf{I}_{N_2})$  represent the AWGNs at the destination antennas and RF-to-baseband transformation at the destination, respectively.

Then the SNR at the destination  $\gamma_D$  from (3) and (7) is obtained as

$$\gamma_D = \frac{P_r \|\mathbf{h}_2\|^2}{\sigma_{a,d}^2 + \sigma_{c,d}^2} = \begin{cases} \frac{\alpha \rho P_s \|\mathbf{h}_1\|^2 \|\mathbf{h}_2\|^2}{\sigma_{a,d}^2 + \sigma_{c,d}^2}, & P_s \|\mathbf{h}_1\|^2 \leq P_{th}, \\ \frac{\alpha \rho P_{th} \|\mathbf{h}_2\|^2}{(\sigma_{a,d}^2 + \sigma_{c,d}^2)}, & P_s \|\mathbf{h}_1\|^2 > P_{th}. \end{cases} \quad (8)$$

TABLE I. SPECIAL CASES OF THE  $\eta-\mu$  FADING DISTRIBUTION [14]

Fading distribution	$\eta$	$\mu$
Nakagami- $m$	$\eta \rightarrow 1$	$\mu = m/2$
Nakagami- $q$ or (Hoyt)	$\eta \rightarrow q^2$	$\mu = 0.25$
Rayleigh	$\eta \rightarrow 0$	$\mu = 0.5$

### B. The $\eta-\mu$ Channel model

The  $\eta-\mu$  fading model is a general fading model and has a few particular cases, viz., Rayleigh, Nakagami- $m$ , and Hoyt [14]. If any of the dual-hop link encounters  $\eta-\mu$  fading, then the probability density function (PDF) of the instantaneous SNR  $\gamma_\ell$  ( $\ell = 1, 2$ ) can be expressed as [14, eq. (3)]

$$f_{\gamma_\ell}(\gamma) = \frac{2\sqrt{\pi} h_\ell^{N_\ell \mu_\ell}}{\Gamma(N_\ell \mu_\ell) H_\ell^{N_\ell \mu_\ell - 0.5}} \left(\frac{\mu_\ell}{\gamma_\ell}\right)^{N_\ell \mu_\ell + 0.5} \gamma^{N_\ell \mu_\ell - 0.5} \times \exp\left(\frac{2\mu_\ell h_\ell}{\gamma_\ell} \gamma\right) I_{N_\ell \mu_\ell - 0.5}\left(2\frac{\mu_\ell H_\ell}{\gamma_\ell} \gamma\right) \quad (9)$$

where  $h_\ell = (2 + \eta_\ell^{-1} + \eta_\ell)/4$ ,  $H_\ell = (\eta_\ell^{-1} - \eta_\ell)/4$ ,  $\Gamma(\cdot)$  indicates the Gamma function, and  $\eta_\ell$  and  $\mu_\ell$  are the fading parameters. Additionally,  $\gamma_\ell$  and  $I_\nu(\cdot)$  denote the average SNR of the  $\ell$ -th link and modified Bessel function of the first kind of  $\nu$ -th order, respectively [14]. Particular cases of the  $\eta-\mu$  fading model are condensed in Table I where  $m$  and  $q$ , respectively, symbolize the fading parameters of the Nakagami- $m$  and Hoyt fading distributions.

## III. PERFORMANCE ANALYSIS

### A. Outage probability analysis

A dual-hop energy harvesting DF relaying system can be in outage when any link (i.e., source-relay or relay-destination) goes in outage. Mathematically, for the considered system, the outage probability can be written by [7]

$$P_{out} = 1 - p_r[\min(\gamma_R, \gamma_D) > \gamma_{th}] \quad (10)$$

where  $\gamma_R$  and  $\gamma_D$  are given by (6) and (8), respectively, and  $p_r[\cdot]$  indicates probability.

### B. Throughput analysis

The achievable throughput of the considered dual-hop energy harvesting DF relaying system in a delay-limited transmission mode is given by [7]

$$\tau = \frac{1 - P_{out}}{2} U. \quad (11)$$

Using (11), the optimal power-splitting ratio  $\rho^*$  and the optimal throughput  $\tau^*$  can easily be obtained numerically with the help of Matlab or Mathematica.

## IV. SPECIAL CASES

The  $\eta-\mu$  fading model has a few particular cases, viz., Rayleigh, Nakagami- $m$ , and Hoyt. Therefore, the throughput and the outage probability expressions for the distinctive cases (i.e., symmetric and asymmetric fading scenarios) can be acquired from (10) and (11) with special parameters. They are condensed in Table II with special parameters.

TABLE II. SPECIAL CASES FROM THE OBTAINED RESULTS FOR  $\eta - \mu$  FADING CHANNELS

First hop/second hop	$\eta_1$	$\mu_1$	$\eta_2$	$\mu_2$
$\eta - \mu$ /Hoyt	$\eta_1$	$\mu_1$	$q_2^2$	0.5
$\eta - \mu$ /Nakagami- $m$	$\eta_1$	$\mu_1$	1	$m_2/2$
$\eta - \mu$ /Rayleigh	$\eta_1$	$\mu_1$	1	0.5
Hoyt/Hoyt	$q_1^2$	0.5	$q_2^2$	0.5
Hoyt/ $\eta - \mu$	$q_1^2$	0.5	$\eta_2$	$\mu_2$
Hoyt/Nakagami- $m$	$q_1^2$	0.5	1	$m_2/2$
Hoyt/Rayleigh	$q_1^2$	0.5	1	0.5
Nakagami- $m$ /Nakagami- $m$	1	$m_1/2$	1	$m_2/2$
Nakagami- $m$ / $\eta - \mu$	1	$m_1/2$	$\eta_2$	$\mu_2$
Nakagami- $m$ /Hoyt	1	$m_1/2$	$q_2^2$	0.5
Nakagami- $m$ /Rayleigh	1	$m_1/2$	1	0.5
Rayleigh/Rayleigh	1	0.5	1	0.5
Rayleigh/ $\eta - \mu$	1	0.5	$\eta_2$	$\mu_2$
Rayleigh/Hoyt	1	0.5	$q_2^2$	0.5
Rayleigh/Nakagami- $m$	1	0.5	1	$m_2/2$

TABLE III. SIMULATION PARAMETERS

Parameter	Value	Parameter	Value
1 $\alpha$	0.9	6 $\sigma_{a,d}^2 = \sigma_{c,d}^2$	0.04W
2 $U$	3	8 $\lambda_1 = \lambda_2$	1W
3 $P_s$	5W	5 $\sigma_{a,r}^2 = \sigma_{c,r}^2$	0.04W

V. NUMERICAL RESULTS AND DISCUSSION

In this section, through Monte-Carlo simulation, the simulation results are drawn to evaluate the performance of the non-linear energy harvester-capable DF relaying system when both links experience  $\eta - \mu$  fading. To evaluate the performance of the considered system, we have numerous choices, for instance, optimal throughput, optimal outage probability, and optimal power-splitting ratio for numerous variable parameters, viz., fading parameters  $\eta$  and  $\mu$ , energy transformation efficiency, noise variances, and antenna arrangements. In circumstances different from the considered, some set of parameters are placed as presented in Table III.

Fig. 1 reveals the outage probability against the power-splitting ratio,  $\rho$ . As expected, growing number of antennas, subsequently enhance the system performance. The outage probability lessens as power-splitting ratio,  $\rho$ , enlarges from 0 to  $\rho^*$  (i.e., an optimal-value of the power-splitting ratio when maximum throughput is obtained), and the outage probability enlarges as the  $\rho$  enlarges from its optimal-value to one.

Fig. 2 shows the throughput with respect to the power-splitting ratio for various saturation threshold power levels. As can be observed, the throughput increases as the level of the saturation threshold power  $P_{th}$  increases. Because enlarging the saturation threshold power level, lessens the possibility of saturation of the energy harvesting receiver, the energy harvesting receiver require additional power to harvest the energy.

In Fig. 3, we showed the outage performance versus the power-splitting ratio for different values of the fading parameter  $\mu$  ( $\mu_1$  and  $\mu_2$ ). From Fig. 3, we observe that the

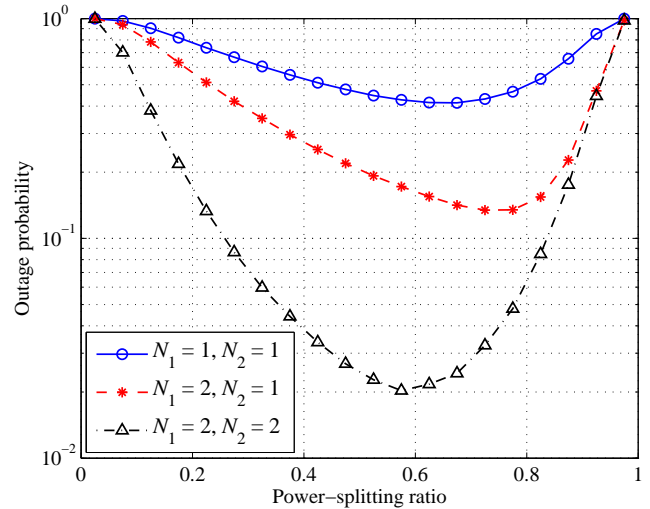


Fig. 1. Outage probability against power-splitting ratio for numerous antenna arrangements when  $P_{th} = 2$ ,  $\eta_1 = \eta_2 = 0.5$ , and  $\mu_1 = \mu_2 = 1$ .

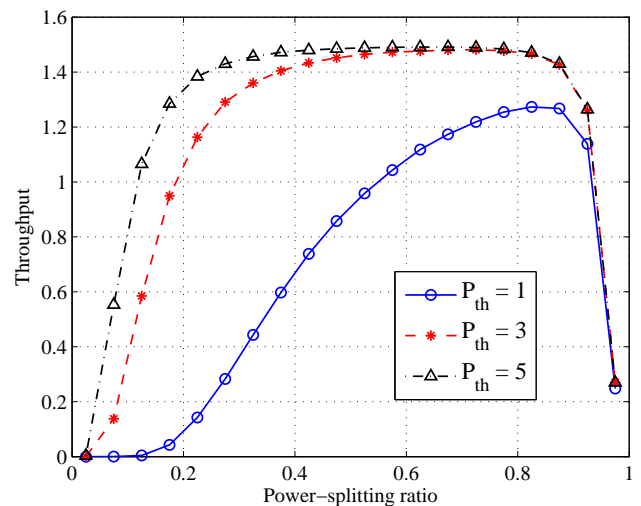


Fig. 2. Throughput as a function of the power-splitting ratio for various values of  $P_{th}$  when  $N_1 = N_2 = 2$ ,  $\mu_1 = \mu_2 = 1$ , and  $\eta_1 = \eta_2 = 0.5$ .

fading parameter  $\mu$  has a notable influence on the system performance. It is also observe that the outage performance is lower at the smaller values of the power-splitting ratio and as well as at the higher values of the power-splitting ratio but outage performance is higher at the medium values the power-splitting ratio.

Fig. 4 presents the outage performance when we vary values of  $\eta$  ( $\eta_1$  and  $\eta_2$ ) and we set  $N_1 = N_2 = 1$ ,  $\mu_1 = \mu_2 = 1$ , and  $P_{th} = 5$ . We observe that the parameter  $\eta$  has a significant relation with the system performance. It is seen that the system achieves better performance when  $\eta_1 > \eta_2$  and the overall performance also increases with increasing the values of the fading parameter  $\eta$ .

ACKNOWLEDGMENT

This research is funded by R&D, Balochistan University of Engineering and Technology, Khuzdar, Pakistan.

REFERENCES

- [1] X. Lu, P. Wang, D. Niyato, and *et al.*, "Wireless networks with RF energy harvesting: A contemporary survey", *IEEE Commun. Surv. Tutorials.*, vol. 17, no. 2, pp. 757–789, 2015.
- [2] A. Hussain, Z. Ahemd, I. Ali, and S.H., Kim, "Energy harvesting relaying network in a delay-tolerant transmission mode over  $\kappa - \mu$  shadowed fading channels", *International Journal of Computer Science and Network Security*, vol. 18, no. 3, pp. 119–125, 2018.
- [3] A. Hussain, Z. Ahmed, U. Rajpoot, and *et al.*, "Energy harvesting relaying network with hardware impairments in  $\eta - \mu$  fading environment", *International Journal of Computer Science and Network Security*, vol. 18, no. 4, 2018.
- [4] A.A. Nasir, X. Zhou, S. Durrani, and R.A. Kennedy, "Throughput and ergodic capacity of wireless energy harvesting based DF relaying network", *proc. IEEE ICC 2014*, Sydney, Australia, June 10–14, 2014.
- [5] O.S. Badarneh, F.S. Almeahmadi, I.S. Ansari, and X. Yang, "Wireless energy harvesting in cooperative decode-and-forward relaying networks over mixed generalized  $\eta - \mu$  and  $\kappa - \mu$  fading channels", *Transaction on Emerging Telecommunication Technology*, DOI: 10.1002/ett.3262, pp.1–18, 2017.
- [6] A. Hussain, S.-H. Kim, and S.-H. Chang, "Dual-hop variable-gain AF relaying with beamforming over  $\kappa - \mu$  shadowed fading channels", *proc. IEEE GLOBECOM 2016*, Washington DC, USA, December 6–8, 2016.
- [7] A. Hussain, S.-H. Kim, and S.-H. Chang, "Non-linear energy harvesting relaying with beamforming and hardware impairments in  $\kappa - \mu$  shadowed fading environment", *Transactions on Emerging Telecommunication Technology*, <https://doi.org/10.1002/ett.3303>, 2018.
- [8] Y. Dong, M.J. Hossain, and J. Cheng, "Performance of wireless powered amplify and forward relaying over Nakagami- $m$  fading channels with nonlinear energy harvester", *IEEE Commun Lett.*, vol.20, no.4, pp.672–675, 2016.
- [9] T.M. Hoang, T.T. Duy, and V.N.Q. Bao, "On the performance of non-linear wirelessly powered partial relay selection networks over Rayleigh fading channels", *proc. 3rd NICS 2016*, Danag, Vietnam, September 14–16, 2016.
- [10] J. Zhang, and G. Pan "Outage analysis of wireless-powered relaying MIMO systems with non-linear energy harvesters and imperfect CSI", *IEEE ACCESS*, vol.4, no., pp.7046–7053, 2016.
- [11] A. Cvetkovic, V. Blagojevic, and P. Ivaniš, "Performance analysis of nonlinear energy-harvesting DF relay system in interference-limited Nakagami- $m$  fading environment", *ETRI Journal*, vol.39, no., pp.803–812, 2017.
- [12] J. Zhang, G. Pan and Y. Xie "Secrecy outage performance for wireless-powered relaying systems with nonlinear energy harvesters", *Front. Inform. Technol. Electron. Engg.*, vol.18, no.2, pp.246–252, 2017.
- [13] K. Wang, Y. Li, Y. Ye, and H. Zhang "Dynamic power splitting schemes for non-Linear EH relaying networks: perfect and imperfect CSI", *Proc. VTC2017-Fall*, Toronto, Canada September 24–27, 2017.
- [14] A. Hussain, S.-H. Kim, and S.-H. Chang, "On the performance of dual-hop variable-gain AF relaying with beamforming over  $\eta - \mu$  fading channels", *IEICE Transactions on Communications*, vol.E100.B, no.4, pp.619–626, 2017.

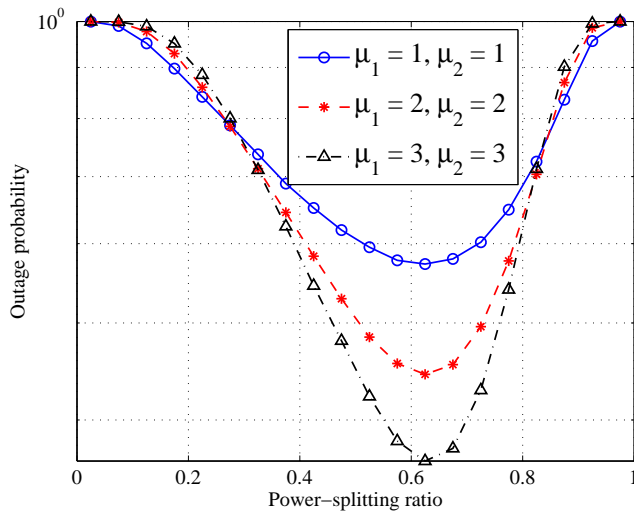


Fig. 3. Throughput versus power splitting ratio for various values of the parameter  $\mu$  (i.e.,  $\mu_1$  or  $\mu_2$ ) when  $P_{th} = 2$ ,  $N_1 = N_2 = 2$ , and  $\eta_1 = \eta_2 = 0.5$ .

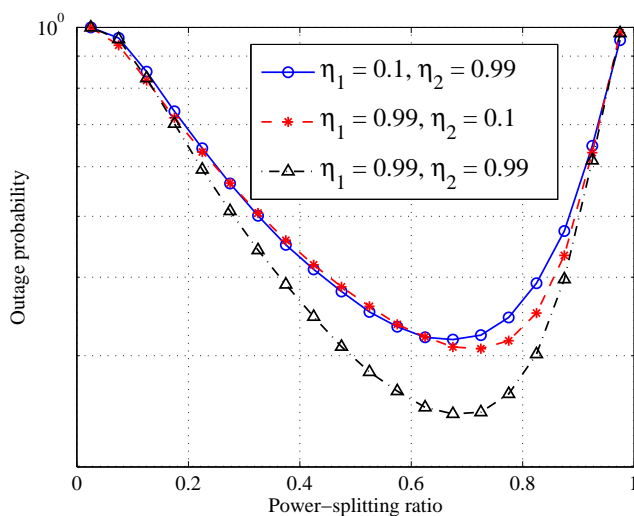


Fig. 4. Outage probability for various values of the fading parameter  $\eta$  (i.e.,  $\eta_1$  or  $\eta_2$ ) when  $N_1 = N_2 = 1$ ,  $\mu_1 = \mu_2 = 1$ , and  $P_{th} = 5$ .

VI. CONCLUSION

In this work, a wireless energy harvesting DF relaying network having multiple-antennas at the destination and source terminals that is based on a practical non-linear model is evaluated over  $\eta - \mu$  fading channels. A PSR protocol and a non-linear energy harvester were considered at the relay node. The system performance were evaluated regarding the throughput and outage probability for numerous set of parameters, such as number of antennas and parameters,  $\eta$  and  $\mu$ . The  $\eta - \mu$  fading model has a few particular cases, viz., Rayleigh, Nakagami- $m$ , and Hoyt. These results are general, therefore, new results were deduced for different fading conditions. The affected system performance by the insignificant saturation threshold power were minimized with the larger number of antennas.